Interested investors and intermediaries: When do ESG concerns lead to ESG performance?

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Keywords: Asset management, investor preferences, asset pricing, corporate social responsibility, impact investing

JEL Classification: G11, G23, G34, M14, M40

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1 Introduction

Investors have allocated vast amounts of wealth to investment intermediaries. Assets under management topped \$85 trillion as of the end of 2017 (Baghai et al., 2018). Because of their holdings, intermediaries can influence firms through voting, voice, and portfolio choices (e.g., Dimson et al., 2015; McCahery et al., 2016).¹ While this influence can be wielded in a disinterested manner to maximize portfolio risk-adjusted returns, evidence suggests that investors and intermediaries have preferences that go beyond cash flows (e.g., Bolton et al., 2020; Gantchev et al., 2018; Friedman, 2020; Li and Raghunandan, 2019).²

In this paper, we study the implications of investors and intermediaries that have preferences over the actions that a firm takes. We focus on a model where a continuum of atomistic investors compete to purchase shares in a firm and can then exert influence efforts that affect the action the firm's manager takes. Despite their preferences, these direct investors optimally exert no influence efforts. Because their holdings are small, the benefits to them are negligible relative to the private cost of influence efforts.

An intermediary, such as an asset manager, makes portfolio decisions on behalf of a measurable fraction of investors, but also has private preferences for the action the manager takes. This preference can be for actions associated with higher expected cash flows (e.g., beneficial governance attributes) or lower expected cash flows (e.g., costly emissions reduc-

¹Asset managers, even passive ones, have a legal duty of care to their investor clients that requires them to vote their clients' proxies (https://www.sec.gov/rules/final/ia-2106.htm#P55_8940). Large asset managers "maintain dedicated investment stewardship teams, which independently develop their own guidelines for engagement and voting." (Mallow, 2019, p. 10)

²Sustainable investment funds topped \$30 trillion as of the beginning of 2018 (GSIA, 2018). Furthermore, the U.K. Financial Reporting Council recently amended its stewardship code to require investment managers to consider environmental, social, and governance (ESG) factors when making investments. While not mandatory, the Financial Conduct Authority requires that asset managers either comply with the stewardship code or explain why they do not (Trentmann, 2019). Institutional investors surveyed in Krueger et al. (2020) incorporate ESG issues in 41% of their portfolios, on average. Kim and Yoon (2020) examine the determinants and implications of active mutual funds' public adoption of the United Nations Principles for Responsible Investment.

tions). The asset manager optimally exerts non-negligible influence efforts, because of both its private interest in the firm's action and its desire to improve the portfolio outcome for its client investors. We find that absent an intermediary, investors' preferences affect neither the firms' action nor the firms' price. However, in the setting with an intermediary, both the investors' and the intermediary's preferences are reflected in corporate actions and stock price.

Interestingly, while the intermediary overcomes the free-rider problem of influence, another free-rider problem among atomistic investors takes its place. When we endogenize the fraction of investors who delegate, we find that all investors prefer to invest directly rather than to delegate. Because the intermediary's influence efforts affect the firm's expected cash flows, direct investors also benefit from the influence activities taken by the intermediary. However, they do not bear the costs of the influence efforts, which are borne only by the investors who delegate their portfolio decisions to the intermediary (i.e., via fees). As a single investor's decision to delegate does not change the intermediary's influence effort, no investors choose to do so in equilibrium. Free riding on the intermediary (and the delegating investors) thus pushes the equilibrium back to that without an intermediary: all investors invest directly in the firm and exert no costly influence efforts. A standard view is that socially responsible funds cater to like-minded investors. Our results highlight that investor preferences over corporate actions and outcomes are insufficient to motivate small investors to allocate funds to interested intermediaries, such as funds focused on sustainable or impact investments.³

We show that an interior degree of delegation arises when there are frictions, such as a fixed cost of direct investment, as in Admati et al. (1994), which could represent the transaction costs direct investors bear and that can be avoided by portfolio delegation.⁴

³In Section 4, we discuss the importance of investor utility from share ownership in and of itself, which can offer a "warm glow" and motivate small investors' portfolio allocations absent other frictions.

⁴Alternatively, the fixed cost of direct investment could represent profits that the intermediary is able to generate from trading on private information (see Marinovic and Varas, 2019) or trading mistakes that direct investors make (e.g., Bushee and Friedman, 2016). We explore these situations in Appendix B and find similar results to the fixed cost of trading.

We find that the fraction of investors who delegate is decreasing in the divergence between the intermediary's and manager's preferred actions. This force provides a justification for an intermediary who is committed to exert ESG-related influence activities choosing to limit their portfolio to firms that already have sufficient ESG performance. In our model, intermediaries benefit from preference alignment because it weakens their incentives for costly influence efforts, which lowers the costs borne by delegating investors. For the same reason, we also find that an intermediary whose preferences are aligned benefits from having strong private preferences over the manager's action, as this further decreases the relative desire to exert influence efforts to increase cash flows. Jointly, these results provide a novel justification for the evidence that intermediaries are concerned about corporate governance (McCahery et al., 2016) but frequently vote with management (e.g., Heath et al., 2021a).⁵ They also provide an economic foundation for ESG funds focusing investment on firms managed by ESG-oriented managers rather than those where ESG-oriented influence would have the greatest effect. Again, the interest alignment allows funds to economize on influence or monitoring activities, consistent with the evidence presented in Heath et al. (2021b).

With endogenous delegation, furthermore, changes in investors' preferences have nonmonotonic effects on the fraction of investors who choose to delegate portfolio decisions to the intermediary. As before, this effect largely operates through the effects of preferences and interests on the magnitude of costly influence efforts undertaken by the intermediary, with fewer investors delegating when the intermediary is expected to exert more costly effort.

Similarly, an increase in the costs of direct investing can increase the firm's stock price. This is a potentially counterintuitive effect, but evolves naturally from the forces in our model. Greater costs increase the fraction of delegating investors. This increases the intermediary's incentive to exert cash-flow increasing influence efforts, which in turn increase cash flows. A decrease in retail investor costs, therefore, can have a negative effect on stock

 $^{^{5}}$ BlackRock, a large asset manager, votes at "more than 17,000 shareholder meetings globally each year, on over 160,000 ballot items," and their "starting position is to support management unless severe governance or performance concerns are identified." (Novick et al., 2018, p. 9)

price because it facilitates free-riding on investor influence efforts.

In two extensions to the main model, we examine the implications of additional forces that may be at play. First, we introduce a second risky asset, which allows us to examine the effect of externalities on the equilibrium. We find that a more positive externality increases the intermediary's effort and, thus, reduces the fraction of investors that choose to delegate. In this setting we find that more investors potentially delegate their trading when the intermediary commits not to trade in assets that experience a positive externality. The reason is that not trading effectively commits the intermediary to exert less influence and, thus, makes delegation less costly.

Finally, we explore the effects of an insider who owns a fixed fraction of the firm, has private interests in the manager's action, and can also exert costly influence efforts. The presence of the insider changes the influence activities of the intermediary. In the Nash equilibrium to the influence subgame, they anticipate each others' influence efforts when choosing their own. Greater holdings by the insider imply greater influence efforts from her, allowing the intermediary to reduce its influence efforts and economize on those costs. Because of this, an increase in the fraction of the firm held by the insider, while reducing the shares held by direct investors, can either increase or decrease the equilibrium holdings of delegating investors. Our last extension also plausibly captures a large index investor who, via its indexing policy, exogenously holds a fixed fraction of a firm's shares.⁶

This study contributes broadly to our understanding of investors' influence efforts, delegation choices, and the effects of preferences over managerial actions. Our framework and results can help inform research and policy-making related to asset management, investor activism, and corporate social responsibility. We contribute to the literatures on investment delegation, blockholder influence, and sustainable or impact investment. While the literature suggests that blockholders can overcome the free-rider problem associated with influence or monitoring (for a review, see Edmans (2014)), we show that, in turn, the free-rider problem

⁶Managers of large index funds, while passively selecting their holdings, do engage in influence activities. See, e.g., Novick et al. (2018).

of delegation prevents intermediaries from becoming blockholders in the first place.

Several recent studies focus on sustainable investment. For example, in Pástor et al. (2021), investors' utility increases in both cash flows as well as in nonpecuniary benefits that investors derive from holding (green) shares. The strength of the utility from nonpecuniary benefits creates a factor that affects returns and an increase to this factor increases the returns from green shares. In our model, the only benefit that investors have from holding shares are cash flows. Instead, we focus on the cash-flow implications of the manager's actions and assume that investors and the intermediary (may) have a preference directly over the action. Green and Roth (2021) use a model to illustrate normative implications of investors' social motivations, and highlight how optimal investment strategies depend on whether social motivations are pure or impure, using the nomenclature of Andreoni (1990). Dyck et al. (2019) show that institutional investors' preferences are associated with portfolio firms' environmental and social actions and policies.

Closely related to our study is Marinovic and Varas (2019) who investigate the dynamic profitability of a blockholder who can influence the firm's managers. Marinovic and Varas (2019) show that asymmetric information about the ability to influence the firm affects the blockholder's incentives to take a positive influence in order to derive gains from trade. Our study differs because we are particularly interested in the impact of preferences on actions and price. In contrast to Marinovic and Varas (2019), we focus on investors' decision to delegate their trading to a blockholder. Related to the delegation decision, Gârleanu and Pedersen (2018) model investor search for intermediaries who may have informational advantages. Gârleanu and Pedersen (2018) show that the search for informed asset managers can yield a similar equilibrium as the search for information. Informed asset managers outperform uninformed ones in expectation, and not all investors should expend costs to find an informed asset manager. Strategic interactions between intermediaries become more complicated in a dynamic game, in which reputation can play a role and intermediaries can coordinate (e.g., Dimson et al., 2019). The next section provides the model setup and main analysis. Extensions are considered in Section 3. In Section 4 we discuss implications, and Section 5 concludes.

2 The Model

There is 1 firm with a manager who takes an action, a. The firm's cash flows per share are given by $x = \beta a + \varepsilon$. We assume that the expected cash flow is $E[x] = \beta a$, where a is the action the manager takes and $\beta > 0$ is the impact of the firm's manager's action on the firm's expected cash flow. The stochastic portion of cash flows, ε , is normally distributed with mean zero and variance σ^2 . We assume that the firm is traded on a competitive market and that the supply of shares equals 1. There is a continuum of risk averse investors of mass 1. Each investor has a CARA utility function with τ as the parameter of absolute risk aversion, such that the aggregate risk aversion in the economy is equal to τ .

The timeline consists of three periods. In the first period, investors trade. In the second period, investors can influence the manager via efforts, m_j , for investor $j \in [0, 1]$. The manager then takes the action a. In period 3, the firm's cash flows are realized and all parties consume.

t = 1	t = 2	t = 3		
trading, p	investor influence, m_j	cash flows, x		
action choices, a				

F	'igure	1:	Time	line
	0			

The manager chooses her action, a, to maximize the following utility function

$$u_m = -\left(a - m - \mu\right)^2,\tag{1}$$

where $m = \int m_j dj$ is the aggregate influence activities, described further below, and μ is the action that the manager would take absent investor influence. This leads to an action choice

$$a = m + \mu. \tag{2}$$

Outside of the monetary utility, we assume that each investor has a preference for the action taken by the firm's manager and that influencing the manager's action comes at a personal cost. As a result, each investor has a certainty equivalent of

$$CE_{j} = q_{j} \left(\beta a - p\right) - \frac{\gamma}{2} \left(a - A_{j}\right)^{2} - \frac{c}{2}m_{j}^{2} - \frac{1}{2}\tau q_{j}^{2}\sigma^{2},$$
(3)

where $\frac{c}{2}m_j^2$ is the cost of influence and $\frac{\gamma}{2}(a-A_j)^2$ captures investors' preferences for, e.g., corporate social responsibility (CSR), governance, or other aspects of the firm not directly reflected in cash flows, x. As a result, the parameters, $\gamma \ge 0$ and c > 0 determine, respectively, investors' costs of the manager's action deviating from their preference and the direct cost of influence activities. A higher γ implies investors care more about the manager's action, all else equal (i.e., relative to the firm's cash flows).

2.1 No intermediaries

Without intermediaries, $m = \int_{j=0}^{1} m_j dj$. The effect of a single investor on the manager's action is negligible, as $\frac{da}{dm_j} = dj \approx 0$. However, the marginal cost from an atomistic investor's perspective is $cm_j \geq 0$. Atomistic investors therefore optimally exert no influence effort, as their efforts have positive cost but infinitesimal benefit because their marginal effects on managerial actions are also atomistic. That is, small investors hold small portfolios and have small effects, even though together they hold the entire firm and could provide a measurable level of influence in aggregate. This captures the fundamental free-rider problem of investor activism. Costly influence efforts yield a social benefit but private costs. It is optimal for each small investor to exert no influence effort, even if they disagree with the actions taken by the manager or can improve cash flows by exerting effort. We summarize this result in Lemma 1 below.

Lemma 1 Atomistic investors find it optimal to exert no influence efforts. Without intermediaries, the firm's price is independent of investors' preference over the manager's action.

Lemma 1 shows that in our setting, investors' non-monetary preferences do not have a price impact as the benchmark price is given by $p^b = \beta \mu - \tau \sigma^2$, expected cash flows minus a risk premium. The reason is that an investor's non-monetary utility is affected by the manager's action regardless of the number of shares that investor demands. As a result, the marginal value of increasing demand is entirely driven by the investor's financial preferences such that, in equilibrium, only financial preferences manifest in price. In our model, this is the case despite the fact that all investors agree on their privately preferred action, A_j .

2.2 An interested intermediary

We next introduce an intermediary who can take costly influence efforts on behalf of its portfolio investors, but also may be interested separately in the manager's action, a. The fraction of investors who delegate to the intermediary is $\lambda \in [0, 1]$. At this stage, we take λ as given, deferring the derivation of an equilibrium λ to later sections.⁷ Without loss of generality, we order the (homogeneous) investors such that investors $j \in [0, \lambda]$ delegate to the intermediary while investors $j \in (\lambda, 1]$ select their portfolios directly.

Conditional on λ investors delegating, the intermediary maximizes the following utility function:

$$u_{i} = \int_{j=0}^{\lambda} CE_{j} di - \frac{\gamma}{2} (a - A_{i})^{2} - \frac{cm_{i}^{2}}{2}$$

$$= \lambda CE_{j} - \frac{\gamma}{2} (a - A_{i})^{2} - \frac{cm_{i}^{2}}{2}, \qquad (4)$$

where $CE_j = q_i (\beta a - p) - \frac{\gamma}{2} (a - A_j)^2 - \frac{1}{2}\tau q_i^2 \sigma^2$ is the certainty equivalent of an investor

⁷The exogenously delegating investors could, for instance, be investors who gain "warm glow" utility from affecting corporate actions and understand that such effects are only feasible via intermediated coordination. Alternatively, they could represent investors with social goals (e.g., pension funds seeking to improve working conditions, religious or university endowments, or sovereign wealth funds).

who delegates their investment choice to the intermediary and (optimally) chooses a personal influence effort of zero, i.e., $m_j = 0$. Furthermore, A_i is the action that the intermediary would prefer the manager to take. Taken together, we assume that the intermediary acts in the interest of all delegating investors but faces a private cost of effort and has preferences over the action that the manager takes.

We next derive optimal influence, share demand, and stock price via backward induction. At t = 2, the interested intermediary has acquired aggregate holdings of q_i in the firm on behalf of investors $j \in [0, \lambda]$ and the price paid to acquire the shares is sunk. Therefore, the intermediary will choose the following influence:

$$m_i^* \in \arg\max_{m_i} \lambda \left(q_i \left(\beta a - p \right) - \frac{\gamma}{2} \left(a - A_j \right)^2 - \frac{1}{2} \tau q_i^2 \sigma^2 \right) - \frac{\gamma}{2} \left(a - A_i \right)^2 - \frac{cm_i^2}{2} da_i^2 \sigma^2 \right)$$

Solving the first-order condition for the globally concave problem yields influence as a function of shares held,

$$m_i^*(q_i) = \frac{\gamma \left(A_i - \mu\right)}{c + (1 + \lambda)\gamma} + \frac{\lambda\gamma \left(A_j - \mu\right)}{c + (1 + \lambda)\gamma} + \frac{\lambda\beta q_i}{c + (1 + \lambda)\gamma}.$$
(5)

The intermediary's optimal influence has three components. The first component is independent of the intermediary's share holdings, q_i , and reflects the intermediary's own preference over the manager's actions. When the intermediary has a preference for higher actions relative to the manager (i.e., $A_i > \mu$), the intermediary will provide more influence effort.

The second component in equation (5) is also independent of the intermediary's share holdings, q_i . However, a higher value of λ increases its weight. This component reflects the investors' preference over the manager's action that the intermediary internalizes because she acts on their behalf. That is, non-monetary preferences by both the intermediary (direct) and the delegating investors (indirect) matter for the intermediary's choice of influence.

Finally, the third component in equation (5) depends on both the fraction of investors who delegate, λ , and on the intermediary's holdings, q_i . This component reflects the cash-flow

implications of influence. When a higher fraction of investors delegate, the intermediary acts on behalf of more shareholders and, therefore, has stronger incentives to increase cash flows. Similarly, a higher q_i increases the incentives to provide influence because the intermediary has a larger benefit from the increased cash flows. Greater efficacy, β , means that every unit of influence has a more positive effect on cash flows, further increasing the benefit of influence efforts to the delegating investors.

Note that the intermediary's influence can be negative. This happens when the intermediary's preference for the manager's action is sufficiently below the manager's preference, i.e., $A_i \ll \mu$. This could, for instance, reflect an intermediary who objects to certain business practices that, while profitable, may be socially undesirable (e.g., related to worker safety or environmental policies).

The coefficient on the cost of influence, c, the coefficient on the preference divergence, γ , and the fraction of delegating investors, λ , deflate the impacts of the additive components in equation (5). However, while an increase in c strictly leads to lower influence effort, an increase in γ or λ makes a divergence in preferences more important. As the importance of the preferences gets large, the intermediary tends to choose an influence level, m_i^* , that induces the manager to choose an effort at the weighted average between the intermediary's and the delegating investors' preferences (i.e., $a \to \frac{1}{1+\lambda} (A_i - \mu) + \frac{\lambda}{1+\lambda} (A_j - \mu)$ as $\gamma \to \infty$). As the intermediary and investors lose interest in the manager's action, i.e., as $\gamma \to 0$, the intermediary shifts to choosing influence efforts only to maximize its investors' cash flows, net of influence costs.

Substituting the intermediary's influence from (5) into (4) yields their objective function at t = 1. Solving the FOC for maximizing the objective over the demand for shares yields

$$q_i^*(p) = \frac{\beta \frac{c\mu + \gamma A_i + \lambda \gamma A_j}{c + (1 + \lambda)\gamma} - p}{\tau \sigma^2 - \frac{\beta^2 \lambda}{c + (1 + \lambda)\gamma}},$$
(6)

which is a maximum for $\tau \sigma^2 - \frac{\beta^2 \lambda}{c + (1 + \lambda)\gamma} > 0$. If $\tau \sigma^2 - \frac{\beta^2 \lambda}{c + (1 + \lambda)\gamma} < 0$, then the intermediary's

expected utility is everywhere increasing in q_i , such that $q_i \to \infty$. In what follows, we assume $\tau \sigma^2 > \frac{\beta^2 \lambda}{c+(1+\lambda)\gamma}$ to focus on the interior solution.

Similar to models without influence, the intermediary will demand fewer shares when the price of the firm, p, is higher, when cash flows are riskier (i.e., when σ^2 is higher), or when the investors that delegate are more risk averse (higher τ). In contrast to models without influence, the numerator in the demand function is not given by the difference between expected cash flows and price. The reason is that with influence, the intermediary's demand itself has an effect on the expected cash flows. This leads to a quadratic effect of q_i on the intermediary's expected cash flows, such that this effect appears as a reduction to the denominator, rather than as an increase to the numerator. Holding price fixed (because we have not yet solved for the equilibrium price), higher demand increases incentives to take an influence action that increases cash flows. Therefore, a higher impact of the influence on cash flows (through the productivity parameter, β) or an interest in higher action (through the parameters A_i and A_j) increase demand. This is counteracted by the cost of effort, c, or the importance of the intermediary's preference, γ , because these two parameters reduce the effect of demand on influence taking. The fraction of investors that delegate has multiple effects (as we discuss above in the derivation of the intermediary's optimal influence), and the impact on the intermediary's demand depends on the relative size of the parameters. The denominator in equation (6) shows that in the setting with an intermediary, the investors' and the intermediary's non-monetary preferences have an impact on their demand of shares beyond their impact on expected cash flows.

Recall that direct investors optimally exert no influence efforts even though they are interested in the manager's action above and beyond its effect on cash flows. Anticipating the intermediary's demand and influence, a direct investor's demand is given by

$$q_j^* = \frac{E\left[x\right] - p}{\tau\sigma^2},\tag{7}$$

where $E[x] = \beta (\mu + m_i^*)$. That is, the demand of a direct trader in a setting where an intermediary takes an influence action takes the same form as in a setting without influence: expected cash flows net of price, divided by the product of risk aversion and cash flow variance. Substituting E[x] and simplifying terms yields

$$q_j^* = \frac{\beta \frac{c\mu + \gamma A_i + \lambda \gamma A_j}{c + (1 + \lambda)\gamma} - p}{\tau \sigma^2 - \frac{\beta^2 \lambda}{c + (1 + \lambda)\gamma}} \Rightarrow q_j^* = q_i^*.$$
(8)

Surprisingly, a direct investor has exactly the same demand as an investor who delegates their portfolio choice to the intermediary. To aid intuition for this equivalence, note that, although a higher cost of effort does not directly change a direct investor's utility, it does change the expected cash flow of the firm. As a result, even a direct investor who does not monitor internalizes the cost of influence.⁸

Price is defined by the market clearing condition, $1 = \lambda q_i^* + (1 - \lambda) q_j^* \Leftrightarrow 1 = q_i^*$. The following proposition characterizes the equilibrium.

Proposition 1 When λ investors delegate to the interested intermediary: 1) the marketclearing price is $p^* = E[x] - \tau \sigma^2 = \beta \frac{c\mu + \gamma A_i + \lambda \gamma A_j + \beta \lambda}{c + (1 + \lambda) \gamma} - \tau \sigma^2$; 2) the influence action taken by the intermediary is $m_i^* = \frac{\beta \lambda + \gamma (A_i - \mu) + \lambda \gamma (A_j - \mu)}{c + (1 + \lambda) \gamma}$; 3) the manager's action is $a^* = \frac{c\mu + \gamma A_i + \lambda \gamma A_j + \beta \lambda}{c + (1 + \lambda) \gamma}$; and 4) all investors hold the same portfolios regardless of delegation, i.e., $q_j^* = q_i^* = 1$.

In equilibrium, the intermediary's influence action is a weighted average of the impact on cash flows (through the productivity of effort, β , and the fraction of delegating investors, λ) and the intermediary's direct and indirect preferences. When the cost of effort increases, the intermediary chooses a lower action. When the importance of preferences increases, the intermediary responds less to the cash flow incentives and more to their preferences. This could increase or decrease the chosen action. Note that the effect of investors' preference parameter, A_j , on managerial actions, a, and stock price, p, is increasing in the fraction of

⁸We show in the appendix that the result that the intermediary's demand per delegating investor is the same as that of a direct investor holds for a general, convex cost of monitoring.

investors who delegate, λ . In our model, intermediaries exert influence effort on behalf of their portfolio investors, while direct investors find it optimal to exert no influence efforts. A greater fraction of delegating investors increases the weight that the intermediary places on investors' preferences relative to the intermediary's preferences and the cost of influence effort. This result is interesting because it implies a potentially counterintuitive result: direct investors' preferences may be better reflected if direct investors play a weaker role in the market, i.e., as λ increases.

We next discuss how the economic features of the setting affect the firm's stock price, using comparative statics.

Corollary 1 The firm's stock price is decreasing in investor risk aversion, τ , and the variance of cash flows, σ^2 . Stock price is increasing in the degree to which the manager's action affects cash flows, β , as well as the manager's, intermediary's, and investors' desired actions, μ , A_i , and A_j , respectively.

As in standard one-period market models with rational risk-averse investors, the price is decreasing in τ and σ^2 . For the effects of the other parameters on price, we focus on their effects on expected cash flows. Expected cash flows are increasing in the effect of the manager's action on cash flows, β , the manager's desired action, μ , and the intermediary's desired action, A_i . Increases in β , A_i , and A_j increase the benefit of inducing the manager to exert higher efforts (for $\beta > 0$ and $\gamma > 0$), because it induces the intermediary to choose more positive influence. A higher value for μ increases expected cash flows because it implies the manager, all else equal, prefers a higher level of productive effort.

Corollary 2 The firm's stock price can be increasing or decreasing in the fraction of delegating investors, λ , and in the cost parameters γ and c. Price is increasing in the fraction of investors that delegate, λ , if β $(c + \gamma) + \gamma^2 (A_j - A_i) + c\gamma (A_j - \mu) > 0$ and decreasing in λ otherwise. Price is increasing in the cost of influence effort, c, if $\gamma (A_i - \mu) + \lambda \gamma (A_j - \mu) + \beta \lambda <$ 0 and decreasing in c otherwise. Price is increasing in the cost of the manager's action deviating from the intermediary's interests, γ , if $c(A_i - \mu) + c\lambda(A_j - \mu) - \beta\lambda(1 + \lambda) > 0$, and decreasing otherwise.

Interestingly, expected cash flows and price can be increasing or decreasing in λ , c, and γ . Increasing the fraction of delegating investors, λ , causes the intermediary to increase its influence because of the positive cash flow effect but can cause the intermediary to reduce its influence when investors prefer a sufficiently small or negative action. Specifically, an increase in λ decreases the intermediary's influence when investors prefer a relatively low managerial action, such that $\beta (c + \gamma) + \gamma^2 (A_j - A_i) + c\gamma (A_j - \mu) < 0 \Leftrightarrow A_j < \frac{\gamma A_i + c\mu}{\gamma + c} - \frac{\beta}{\gamma}$. When the investors' non-monetary preferences outweigh their monetary preferences, an increase in λ reduces price.

When $\gamma (A_i - \mu) + \lambda \gamma (A_j - \mu) + \beta \lambda > 0$, price is decreasing in c, because a higher cost of influence causes the intermediary to choose a lower level of influence. However, increasing c increases cash flows when $\gamma (A_i - \mu) + \lambda \gamma (A_j - \mu) + \beta \lambda < 0$, which requires $\frac{A_i + \lambda A_j}{1 + \lambda} < \mu$. In this scenario, the intermediary's preferences cause it to exert negative influence efforts, $m_i^* < 0$, which lower the manager's effort and, thus, lower cash flows. In such a case, an increase in the cost of influence causes the intermediary to reduce the magnitude of influence, which implies moving m_i^* closer to zero by increasing it.

For the effects of γ , $c(A_i - \mu) + c\lambda (A_j - \mu) - \beta\lambda (1 + \lambda) > 0$ is equivalent to $\frac{A_i + \lambda A_j}{1 + \lambda} - \mu > \frac{\beta\lambda}{c}$, which implies the intermediary's preferred action, conditional on delegation, λ , is higher than the manager's. An increase in γ causes the intermediary to care more about this divergence, which results in a more positive influence effort, an increase in the manager's action, and higher cash flows and price. In contrast, if $c(A_i - \mu) + c\lambda (A_j - \mu) - \beta\lambda (1 + \lambda) < 0 \iff \frac{A_i + \lambda A_j}{1 + \lambda} - \mu < \frac{\beta\lambda}{c}$, an increase in γ will result in a lower stock price because for this range of parameters an increase in γ causes the intermediary to choose a more negative level of influence. When $\frac{A_i + \lambda A_j}{1 + \lambda} < \mu$, this is straightforward, as the intermediary prefers for the manager to take a lower action. For $\frac{A_i + \lambda A_j}{1 + \lambda} \in (\mu, \mu + \beta\lambda/c)$, the intermediary's private

preference for managerial action is greater than the manager's. However, it remains lower than the action the manager would take in the presence of an intermediary and investors who have no private preference ($\gamma = 0$), ceteris paribus. That is, the intermediary values the effect of influence on expected cash flows, which pushes the manager's optimal action up to $\mu + \beta \lambda/c$. However, this is higher than the intermediary's private preference, so an increase in γ causes the intermediary to exert a more negative influence effort.

Now, assume that investors who delegate to the intermediary must compensate the intermediary for their efforts (e.g., via asset management fees). Technically, this is similar to bringing in a participation constraint for the intermediary. With such fees equally split across delegating investors, we have

$$CE_i^* = \frac{u_i}{\lambda} = \frac{1}{2}\tau\sigma^2 - \frac{1}{2}\frac{\beta^2\lambda^2 + \lambda\gamma c\left(\mu - A_j\right)^2 + \lambda\gamma^2\left(A_i - A_j\right)^2 + c\gamma\left(\mu - A_i\right)^2}{\lambda\left(c + \gamma + \lambda\gamma\right)}.$$
 (9)

For direct investors, the equilibrium certainty-equivalent is

$$CE_{j}^{*} = \frac{1}{2}\tau\sigma^{2} - \frac{1}{2}\gamma \frac{(\beta\lambda + c(\mu - A_{j}) + \gamma(A_{i} - A_{j}))^{2}}{(c + \gamma + \lambda\gamma)^{2}}.$$
(10)

Note that delegating and direct investors both capture the same risk premium, $\frac{1}{2}\tau\sigma^2$, as in a model without influence. In addition, both types of investors suffer from a difference between their preferred action and the manager's equilibrium action.

Absent non-monetary preferences (i.e., when $\gamma = 0$), direct investors have a certainty equivalent that is the same as in a model without influence, $CE_j^*|_{\gamma=0} = \frac{1}{2}\tau\sigma^2$. Delegating investors, however, still have to pay for the equilibrium influence costs that the intermediary undertakes when motivated only by their effects on cash flows, implying $CE_i^*|_{\gamma=0} = \frac{1}{2}\tau\sigma^2 - \beta^2\frac{\lambda}{c}$. A preference disalignment either between manager and intermediary or between manager and investors causes the intermediary to choose a higher influence action. Because the cash-flow effects are priced in, this higher action is costly to the intermediary and, therefore, to the delegating investors. As a result, the delegating investors are better off when the fund manager's preferences align with those of the corporate manager, i.e., when $A_i \rightarrow \mu$.

Corollary 3 All else equal, the intermediary and delegating investors are better off when the fund manager's preferences are more closely aligned with the manager's, i.e., when $(\mu - A_i)^2$ is small.

Many types of funds operate without preferences over managerial actions above and beyond cash flows. However, such preferences can be beneficial in equilibrium if $\frac{dCE_i^*}{d\gamma} > 0$. However, when the intermediary's and investors preferences are aligned with managers, $A_i = A_j = \mu$, then delegating investors benefit from stronger non-monetary preferences, γ , as they reduce the intermediary's incentive to provide cash-flow increasing influence. In such a situation, an increase in delegation causes the intermediary to choose a higher action (to increase cash flows), which reduces expected utilities of delegating investors. An increase in delegation has two additional effects. First, as we discuss above, an increase in λ causes the intermediary put less weight on its own non-monetary preferences. The effect on influence depends on parameter values. Finally, more delegation allows the costs of influence to be spread over more investors, which increases delegating investors' expected utilities.

A comparison of (9) and (10) yields $CE_i^* \leq CE_j^*$, as only the delegating investors bear the costs of influence activities and are assumed to compensate the intermediary for it's γ -related disutility. The ordering of certainty equivalents between delegating and direct investors implies that each investor is better off investing directly rather than through the intermediary. The intermediary's actions are foreseeable for a given λ and are thus impounded into price in such a way as to stop the intermediary (and the delegating investors) from ex-ante benefitting from the ability to take them. Furthermore, investors hold the same portfolios regardless of whether they invest directly or through the intermediary.

Proposition 2 Absent additional frictions, no investors would delegate to the intermediary.

Earlier, we highlighted the free-rider problem that causes influence efforts to go to zero in the setting without the intermediary. From an atomistic investor's perspective, influence is materially costly, but the benefits to a single atomistic investor are negligible despite aggregate benefits. The intermediary can act on behalf of a set of investors, thus overcoming the free-rider problem amongst the delegating investors. However, our results thus far show that a free-rider problem remains. The direct investors benefit from the intermediary's efforts and, because they invest directly, can avoid having to pay the intermediary. This free-rider problem eliminates the benefit from delegating investment. This result stands in contrast to the literature on blockholder monitoring that suggests that blockholders overcome the free-rider problem of monitoring. Our analysis points out that, in effect, one free-rider problem merely gets substituted for another. In the next section, we introduce a cost to direct investment to arrive at an interior level of equilibrium delegation and examine the effects of endogenous delegation on our previously derived results.

2.3 Endogenous delegation

We now assume the direct investors pay a (net) transaction cost of κ to participate in the market. As in Admati et al. (1994), the intermediary also bears a cost κ , but this can be viewed as negligible when spread across any measurable mass of delegating investors. The certainty-equivalent utilities are given by CE_i^* and $CE_j^* - \kappa$, where CE_i^* and CE_j^* are defined in (9) and (10), respectively.

The equilibrium degree of delegation, λ , is defined by the solution to $CE_j^* - \kappa = CE_i^*$. Substituting from (9) and (10) and simplifying yields

$$\kappa = \frac{1}{2\lambda} \left(\gamma \left(a^* - A_i \right)^2 + c \left(m_i^* \right)^2 \right), \tag{11}$$

which equates the cost of direct investing with the cost of investing via the intermediary. Note that the investors' preferences affect delegation in (11) only via its effects on the intermediary's optimal influence, m_i^* , and the manager's optimal action, a^* . We next turn to the effects of investors' preferences on the optimal fraction of delegating investors. **Proposition 3** The fraction of investors who delegate is decreasing in A_j for $A_j > \frac{c\gamma\mu + \gamma^2 A_i - c\beta - \beta\gamma}{\gamma(c+\gamma)}$. Otherwise, it is increasing in A_j .

Proposition 3 provides the result that the fraction of investors who delegate decreases in the investors' preference parameter, A_j , when A_j is high. In other words, when investors have strong preferences for the manager's action, fewer investors will delegate *even though* only the intermediary will move the agent's effort closer to the investors' preference. The reason is that when the intermediary increases their influence in response to a higher A_j , the cost imposed on delegating investors increases such that fewer investors delegate. When the investors' preference is sufficiently low, however, a greater weight on A_j (and a lower relative weight on A_i) reduces the intermediary's effort, which causes more investors to delegate.

Note that an increase in A_j can have opposing effects on the manager's action: it increases a through the intermediary's incentives but it decreases a because fewer investors delegate. We analyze the effect of A_j on the manager's action in the following proposition.

Proposition 4 When investors care about the manager's action, the manager's action decreases in A_j when a sufficiently large fraction of investors delegate and increases when fewer investors delegate.

Proposition 4 shows that when the cost of investing directly, κ , is sufficiently low, the manager's action decreases in A_j because the indirect effect dominates: the cost of higher influence causes fewer investors to delegate. This leads to a potentially single-peaked effect of A_j on a. While higher investor preferences initially increase the manager's action, eventually they will decrease the manager's action. Proposition 4 provides the result that changes in investor preferences need not be well reflected in stock price changes. Note that this result is specific to a setting where investor preferences are reflected in price through delegation to an intermediary who, in turn, can affect the manager's decisions.

While our assumption of costs to direct investing seems ad hoc, we can recover similar results by assuming either that the intermediary has pre-trade private information (for example about its efficacy of influence taking) or that direct investors make trading errors and are aware of this possibility before they decide about delegating capital. In the appendix we provide detailed analyses of these two cases. Similar to Marinovic and Varas (2019) we first assume that the intermediary observes a random shock to its cost of influence, such that the cost is $\frac{c}{2}m_i^2 - ym_i$, with $y \sim N(0, \sigma_y^2)$. In the second setting, we assume that direct investors react to noisy, idiosyncratic signals, $y_j \sim N(0, \sigma_j^2)$, as if they are informative about cash flows. In both settings, an increase in the variance of the random variable (σ_y^2 and σ_j^2 , respectively) leads to an increase in the intermediary's expected share holdings, which results in increases in the expected influence effort, managerial action, and cash flows.

3 Extensions

In this section we extend the main model in two directions. In the first extension, we include an additional firm and assume that the action taken by the original firm's manager affects the cash flows of the second firm, i.e., there are cross-firm externalities to influence activities. Second, we assume that there exists an insider in the firm who holds a significant fraction of shares and has a preference over the manager's action as well. The insider could also represent an index fund whose portfolio is relatively fixed by the index it tracks but, due to fiduciary requirements, remains involved in the firm's governance via stewardship, monitoring, or engagement activities. In both extensions, we assume that investors themselves do not have preferences over the manager's action. This allows us to have cleaner expressions for the costs and benefits of delegation.

3.1 Multiple firms

In this section, we add a second risky asset. This allows us to study diversification, crossasset externalities, and self-imposed portfolio constraints (e.g., restrictions on direct holdings of some assets such as shares of oil and gas firms). Specifically, we assume that there are two firms, $f \in \{1, 2\}$, with cash flows of $x_f = \beta_f a_1 + \varepsilon_f$, $Var [\varepsilon_f] = \sigma_f^2$, and $Cov [\varepsilon_1, \varepsilon_2] = \rho \sigma_1 \sigma_2$. The action of firm 1's manager has an externality of $\beta_2 a_1$ on firm 2's cash flows. We assume that $\beta_1 > 0$ and $\beta_1 + \beta_2 > 0$ such that the effects of the manager's action on total cash flows are positive, but the externality can be positive or negative, $\beta_2 \in (-\beta_1, \infty)$. For simplicity of exposition, we suppress firm 2's manager, as illustrating the economic forces of interest requires only one source of cross-firm externalities. As before, (i) both firms are traded on a competitive market, (ii) the supply of shares of each firm equals 1, and (iii) there is a continuum of risk-averse investors of mass 1, each with a CARA utility function such that the aggregate risk aversion in the economy is equal to τ .

Because firm 1's situation is unchanged, the firm's manager continues to choose the action $a_1 = m_1 + \mu_1$. Each direct investor's certainty equivalent is given by

$$CE_{j} = q_{j1}\left(\beta_{1}a_{1} - p_{1}\right) + q_{j2}\left(\beta_{2}a_{1} - p_{2}\right) - \frac{1}{2}\tau\left(q_{j1}^{2}\sigma_{1}^{2} + q_{j2}^{2}\sigma_{2}^{2} + 2\rho q_{j1}q_{j2}\sigma_{1}\sigma_{2}\right),$$
(12)

where we omit the cost of influence because, as discussed above, direct investors choose not to influence the manager. Similar to our main analysis, a fraction $\lambda \in [0, 1]$ of investors delegate their investment decision to the intermediary, which maximizes the utility function in (4) expanded to two firms. Optimizing over influence as well as over quantities leads to prices of $p_f = \beta_f a_1^* - \tau \left(\sigma_f^2 + \rho \sigma_1 \sigma_2\right)$. Interestingly, the externality has no effect on both the risk premium and on the intermediary's demand, such that delegating and direct investors continue to hold the same portfolio. The optimal action and influence are given by $a_1^*(m_i^*) = \mu_1 + m_i^*$ and $m_i^*(q_{i1}^*, q_{i2}^*) = \frac{\gamma(A-\mu_1)}{c+\gamma} + \frac{\lambda}{c+\gamma}(\beta_1 + \beta_2)$. That is, the holdings in firm 2 provide incentives to the intermediary to provide influence in firm 1 because of the externality captured by β_2 . When the externality increases cash flows, the intermediary provides more influence and, thus, increases cash flows for both firms. Interestingly, a higher externality makes delegating investors worse off. While expected cash flows are priced, the expected costs of influence are borne only by the delegating investors.

With endogenous delegation, a higher externality, β_2 , leads to a smaller fraction of delegating investors. The reason is that the free-rider problem of delegation is more severe when influence has a more beneficial impact on the cash flows of the portfolio. In other words, an intermediary may benefit from investing in assets that have negative externalities among each other because they can reduce the magnitude of costly influence activities optimally exerted after investment positions are taken.

The intermediary may also benefit from a commitment to *not* invest in an asset that has a positive externality on other assets that the intermediary will hold. To investigate this, assume that the intermediary commits not to invest in firm $2.^9$ In this situation, the intermediary's and the direct investors' demands are given by

$$q_{i1}^{\dagger} = 1 + \rho \frac{\sigma_2}{\sigma_1},$$
 (13)

$$q_{j1}^{\dagger} = 1 - \rho \frac{\sigma_2}{\sigma_1} \frac{\lambda}{1 - \lambda}$$
, and (14)

$$q_{j2}^{\dagger} = \frac{1}{1-\lambda}.$$
 (15)

In contrast to our previously-derived results, the demands are no longer identical. While direct investors' demand for asset 2 influences their holdings in asset 1, this is not the case for the intermediary.

The heterogeneity in demands has an effect on the intermediary's incentives to exert influence, such that influence can increase as a result of a commitment not to invest in asset 2. In equilibrium, the direct investors hold all shares in asset 2. When ρ is positive, the correlated risk across assets reduces direct investors' willingness to hold shares in asset 1, which in turn implies that the intermediary holds more shares in asset 1 and, thus, has a higher incentive to exert influence. Specifically, the intermediary's optimal influence with

⁹In our model, firm 1 is the focal firm where the intermediary can exert influence and has a preference for the manager's actions. For that reason, we investigate only the commitment not to invest in firm 2.

the commitment not to invest in asset 2 is given by $m_i^{\dagger}\left(q_{i1}^{\dagger}, q_{i2}^{\dagger}\right) = \frac{\gamma(A-\mu_1)}{c+\gamma} + \frac{\lambda\beta_1}{c+\gamma}\left(1 + \rho\frac{\sigma_2}{\sigma_1}\right)$ and the difference in influence is given by

$$m_{i}^{\dagger}\left(q_{i1}^{\dagger}, q_{i2}^{\dagger}\right) - m_{i}^{*}\left(q_{i1}^{*}, q_{i2}^{*}\right) = \frac{\lambda}{\sigma_{1}\left(c+\gamma\right)}\left(\rho\beta_{1}\sigma_{2} - \sigma_{1}\beta_{2}\right).$$
(16)

This difference is positive when $\rho_{\sigma_1}^{\sigma_2} > \frac{\beta_2}{\beta_1}$. A higher value of the left-hand side makes it less attractive for direct investors to hold shares in asset 1, which implies that the intermediary will hold more of these shares. This, in turn, increases the incentives to exert influence and, thus, reduces the benefit of a commitment not to invest in asset 2. The right-hand side shows the relative effort incentive from holding 1 share in asset 2, the higher this is, the more beneficial it is to commit not to invest in asset 2.

The equilibrium prices of the two assets are given by

$$p_1 = \beta_1 \left(\frac{A_i \gamma + c\mu}{c + \gamma} + \frac{\lambda \beta_1}{c + \gamma} \left(1 + \rho \frac{\sigma_2}{\sigma_1} \right) \right) - \tau \left(\sigma_1^2 + \rho \sigma_1 \sigma_2 \right)$$
(17)

$$p_2 = \beta_2 \left(\frac{A_i \gamma + c\mu}{c + \gamma} + \frac{\lambda \beta_1}{c + \gamma} \left(1 + \rho \frac{\sigma_2}{\sigma_1} \right) \right) - \tau \left(\frac{1 - \lambda \rho^2}{1 - \lambda} \sigma_2^2 + \rho \sigma_1 \sigma_2 \right)$$
(18)

Note that β_2 no longer affects p_1 . As a result, the intermediary's expected utility is independent of β_2 . Because the expected cash flows of asset 2 are perfectly priced, the expected utility of direct investors is also independent of β_2 . As a result, changes in β_2 do not affect the fraction of investors that delegate. This leads to the following Proposition.

Proposition 5 In the economy with two risky assets, the fraction of investors who delegate increases when the intermediary commits not to trade in asset 2 for sufficiently large β_2 and sufficiently large κ .

The proposition shows that it can be beneficial for the intermediary to limit the shares that they invest in. For $\beta_2 >> 0$, the commitment not to invest in asset 2 is effectively a commitment to exert less influence effort. This provides an interesting countervailing effect relative to the traditional intuition that investors benefit from holding a portfolio of firms that have positive externalities on each other (abstracting from systematic risk considerations).

3.2 An interested insider

In this extension, we introduce an interested insider into the model. The insider has a fixed and measurable endowment of shares, q_h , and does not participate in the stock market. The insider can, however, exert influence efforts. An alternative interpretation is that the insider has a fixed endowment of shares because she represents a passive index-tracking fund whose stock ownership is determined by the weight of the firm in the index.¹⁰

Let the insider's certainty-equivalent utility be defined as

$$CE_{h} = q_{h}(\beta a) - \frac{1}{2}\tau q_{h}^{2}\sigma^{2} - \frac{\gamma}{2}(a - A_{h})^{2} - \frac{c}{2}m_{h}^{2}.$$

The risk aversion term, $\frac{1}{2}\tau q_h^2 \sigma^2$, will not be affected by any actions the insider takes, but her influence efforts and utility will depend on her share endowment, q_h , and preference parameter, A_h .

Lemma 2 In the presence of an interested insider who holds q_h shares and prefers for the manager to take action A_h , the optimal influence efforts conditional on holdings are

$$m_{h,IN}^{*}(q_{i},q_{h}) = \gamma \frac{\gamma \left(A_{h}-A_{i}\right)+c \left(A_{h}-\mu\right)}{2c\gamma+c^{2}} + \beta \frac{q_{h}\left(c+\gamma\right)-\lambda\gamma q_{i}}{2c\gamma+c^{2}}, \text{ and}$$
$$m_{i,IN}^{*}\left(q_{i},q_{h}\right) = \gamma \frac{\gamma \left(A_{i}-A_{h}\right)+c \left(A_{i}-\mu\right)}{2c\gamma+c^{2}} + \beta \frac{\lambda q_{i}\left(c+\gamma\right)-\gamma q_{h}}{2c\gamma+c^{2}}.$$

The influence effort taken by the insider is increasing in A_h and her shareholdings, q_h , but

¹⁰Managers of index funds undertake significant influence activities, often referred to as "engagement" by practitioners. Mallow (2019) and Novick et al. (2018) discuss large asset managers' engagement activities and preferences in depth. Appel et al. (2016) opens with the following quote from F. William McNabb III, Chairman and CEO of the Vanguard funds, "We're going to hold your stock when you hit your quarterly earnings target. And we'll hold it when you don't. We're going to hold your stock if we like you. And if we don't. We're going to hold your stock when everyone else is running for the exits. That is precisely why we care so much about good governance."

decreasing in the intermediary's preference parameter, A_i and shareholdings, q_i . Similarly, the insider's influence effort is decreasing in the insider's preference parameter and shareholdings. These relations hold because the insider and intermediary can rationally anticipate how each others' holdings and preferences will affect the influence efforts both take. Because the manager's action is the sum of the influence efforts and her preference parameter, μ , the influence efforts are strategic substitutes.

Note that in the two-asset setting, higher shareholdings by direct investors decrease the intermediary's influence because they reduce the intermediary's shareholdings. In the setting with an interested insider, increasing the insider's holdings may reduce the intermediary's holdings. However, the insider's holdings also directly reduce the intermediary's influence because efforts are strategic substitutes. Note, too, that the influence undertaken by the insider does not disappear if her holdings go to zero. Rather, this also requires $A_h = \frac{1}{c+\gamma} (c\mu + \gamma A_i + \beta \lambda q_i)$, which will not generally be satisfied.

The optimal action, conditional on shares held, is

$$a^{*}(q_{i}, q_{h}) = \frac{c\mu + \gamma A_{h} + \gamma A_{i} + \beta q_{h} + \beta \lambda q_{i}}{c + 2\gamma}$$

This is increasing in the intermediary and insiders' preference parameters, in the shares they hold, and in the fraction of delegating investors.

Proposition 6 An increase in the insider's private preference parameter, A_h , is associated with more influence effort from the insider, higher expected cash flows, higher stock price, fewer shares held by direct investors, and more shares held by delegating shareholders.

An increase in the insider's private preference, A_h , naturally leads to more positive influence effort from the insider, which leads to a higher action from the manager, and thus higher expected cash flows and stock price. This allows for the intermediary to economize on its influence efforts, making holdings less costly. This tilts the shareholder base towards the intermediary and delegating investors, even for a fixed λ , although this effect is mitigated if preferences are aligned between the intermediary and the insider, i.e., if $A_h = A_i$. The presence of an interested insider can benefit the intermediary and the delegating investors precisely by providing a mechanism whereby the intermediary can economize on costly influence efforts whose benefits accrue to shareholders more broadly.

Proposition 7 An increase in the insider's share holdings, q_h , is associated with

- (i) higher stock price and fewer shares held by direct investors;
- (*ii*) more influence effort from the insider;
- (iii) lower managerial action, a_{IN}^* , and lower expected cash flows if and only if $0 < \beta^2 \gamma^2 (1 \lambda) < c\tau \sigma^2 (c + 2\gamma)^2 \frac{\gamma(\lambda c 2\gamma)}{\lambda(\gamma(1 c 2\gamma) + (c + 2\gamma)^2)}$; and
- (iv) delegating shareholders holding more shares if and only if $\beta^2 \gamma^2 (1-\lambda) > c \tau \sigma^2 (c+2\gamma)^2$.

When the insider owns more shares, there are fewer shares available for trading. This decreases the number of shares held by direct investors and decreases the risk that is priced on the open market, which in turn increases the firm's price. Because the insider's utility is more affected by the firm's cash flows when q_h increases, she exerts more influence. However, because influence efforts are strategic substitutes, the intermediary will reduce their influence (holding q_i fixed) such that the managerial action and cash flows can decline. In our model, there is a third, indirect, effect. The reduction in effort that the intermediary supplies reduces the intermediary's marginal cost of holding shares. This, in turn, can increase the intermediary's demand. When the intermediary's holdings increase sufficiently, their influence does not decrease as much, such that managerial action and cash flows increase. Note that $c\tau\sigma^2 (c+2\gamma)^2 \frac{\gamma(\lambda - c-2\gamma)}{\lambda(\gamma(1-c-2\gamma)+(c+2\gamma)^2)} < c\tau\sigma^2 (c+2\gamma)^2 \Leftrightarrow (c+2\gamma)(\gamma + c\lambda + \lambda\gamma) > 0$, so there exists a parameter region where the intermediary's holdings increase and the managerial action decreases.

4 Implications

Three themes appear repeatedly across our baseline model and extensions. First, in our setting with preferences over outcomes or actions, investor preferences are not reflected in actions or prices unless investors can overcome the free riding problems. Second, free riding on influence efforts makes delegating investments to an intermediary who can take influence actions less desirable. Indeed, no investors would delegate in the absence of additional frictions (e.g., costs of direct investment, information asymmetry, or behavioral biases that lead to trading errors). Third, delegating investors benefit from mechanisms that allow the intermediary to commit to exert less effort, given that the costs of effort are passed through to the delegating investors. In the discussion below, we adopt an institutional perspective whereby the intermediary is better off when more investors delegate, consistent with investment intermediaries competing for fund flows. With this perspective, our results imply that the intermediary is better off with commitment devices that facilitate smaller influence efforts and thus greater delegation.

Corollary 2 shows that stock prices can increase or decrease when the fraction of delegating investors increases. The change in price occurs because the intermediary is a mechanism whereby investors can overcome the free-rider problem on influence efforts. An increase in the fraction of delegating investors causes the intermediary to care more about investors' preferences and about the firm's cash flows. When investors have preferences for actions that are sufficiently lower than the positive cash flow impact and the intermediary's preferred action, a higher fraction of delegating investors causes the intermediary to exert less influence. Corollary 2 further suggests that lowering the cost of direct investment can, by reducing the intermediary's influence efforts, lead to lower cash flows and stock prices. This suggests a potential negative social effect of intense fee competition between retail brokerages.

In Corollary 3 we show that the intermediary can benefit from high costs of preference misalignment, γ , when the intermediary's preferences are aligned with those of the manager. High costs combined with preference alignment cause the intermediary to prefer influence efforts closer to 0, even when more positive influence leads to higher cash flows. This result provides a novel justification for the joint occurrence of intermediaries being greatly concerned about corporate governance (McCahery et al., 2016) while frequently voting with management (e.g., Heath et al., 2021a).

Corollary 3 also provides an economic foundation for ESG funds preferring to invest in firms managed by ESG-oriented managers. In our model, both managers and investors benefit from this alignment. Managers benefit from having reduced pressure from intermediaries (unmodeled but straightforward to add). Investors benefit from the reduced influence effort costs. In some sense, the matching between funds and firms allows funds to economize on influence or monitoring activities. This is consistent with the result in Heath et al. (2021b) that interested funds tend to choose portfolio firms with aligned interests, which results in less influence activities being undertaken.

An important question is why small investors allocate their portfolios to investments that are aligned with their interests (e.g., funds focused on environmental sustainability). Previous studies have highlighted warm glow as a potential mechanism (e.g., Fama and French, 2007; Friedman and Heinle, 2016), as warm glow leads investors to value their ownership per se. We show that investor interest in corporate actions, rather than warm glow from ownership, is not sufficient to overcome the free-rider problems on influence or delegation, even if preferences are correlated across investors. Alignment of preferences on outcomes between funds and shareholders does not in and of itself lead to greater delegation or corporate actions more aligned with shareholder preferences. Put another way, green funds would not attract green investors if investors care about outcomes (i.e., managers' environmental actions) and understand the free-rider problem. For small investors to tilt their portfolios towards "responsible," "sustainable," or "impact" assets, it is necessary that these investors value such investments in and of themselves or that investors feel stronger about the outcomes when they hold more shares. Using the nomenclature of Andreoni (1990), impure altruism manifested via warm glow may cause outcomes to be more strongly related to investor interests, relative to pure altruism that captures a desire for better outcomes in and of themselves.

In our model, the intermediary internalizes the direct investors' preferences for corporate action. When investors are interested, delegation to the intermediary allows direct investors to overcome the free-rider problem on their efforts and therefore increases the tie between investors' preferences and the manager's equilibrium action. However, this causes the intermediary to exert more influence efforts, which increases costs and deters delegation. Proposition 3 shows that delegation is first increasing then decreasing in investors' preference parameter, A_j . In other words, more positive investor preferences can backfire and lead to lower managerial action due to the effect on investor delegation and the intermediary's influence. This suggests that we should observe a sharp increase in delegation, and potentially a consequent rise in prevalence of ESG funds, as investors begin to be interested, in aggregate, in ESG performance. Delegation to interested intermediaries should level off as the costs of the intermediary's influence get large, driven by investor preferences for very positive managerial actions.

Funds frequently use exclusionary screens (GSIA, 2018), e.g., sustainability focused funds avoiding investments in strip-mining firms to attract similarly-interested investors.¹¹ Although exclusionary screens may be motivated by warm glow, we provide an alternative justification for exclusionary screens that derives from externalities between firms (Section 3.1). Positive externalities of one firm's manager's action on another firm's cash flows can enhance the intermediary's incentive to exert influence, which, while beneficial for cash flows, can make delegating investors worse off relative to direct investors. The intermediary can be better off if they ex-ante commit not to invest in firms on which their focal firms' actions have positive externalities (Proposition 5). As before, the mechanism is a commitment to exert lower influence efforts.

¹¹Alternatively, small investors might tilt their portfolios due to a (mistaken) belief that their small holdings make a difference, i.e., believing that $\frac{da}{dm_j} > 0$. Through this mechanism, negative investment screens and divestment could attract investors who derive utility from beleiving that their investment choices harm bad actors by increasing their cost of capital or lowering firm value (e.g., Teoh et al., 1999).

Negative cash flow externalities reduce influence incentives, which can be beneficial for delegating investors. In a multi-industry economy, delegating investors benefit from intermediaries curtailing influence activities that can improve one portfolio firm's cash flows at the expense of harming those of another portfolio firm. Our model shows that the intermediary, in turn, benefits from including in its portfolio the firms that experience a negative externality. Diversification across industries whose influencable actions have negative externalities on each other can thus provide an additional benefit to funds via an implicit commitment to curtail costly influence actions.

An important feature in our model is that small investors do not have a measurable impact on corporate policies. In contrast, the insider or indexer modeled in Section 3.2, by virtue of their size, is able to influence corporate actions. The insider's influence also allows the intermediary to economize on influence actions, because influence actions are strategic substitutes in improving cash flows. Furthermore, an increase in the insider/indexer's holdings affects the intermediary's equilibrium holdings and efforts and the firm's cash flows, which may help explain existing mixed results on the net effects of index ownership (e.g., Schmidt and Fahlenbrach (2017), Heath et al. (2021a), and Appel et al. (2016)).

These mixed results are also related to how one should interpret the secular shift from active to passive management, with huge sums now invested in index-tracking funds (Segal, 2019).¹² On the one hand, passively allocated funds could be interpreted as intermediaries who have committed not to take any costly influence. They help delegating investors avoid the transaction costs of Section 2.3 while refraining from imposing influence costs on their clients. On the other hand, even passive fund managers participate in influence actions such as proxy voting and other stewardship activities (Mallow, 2019; Novick et al., 2018). This is the interpretation offered in Section 3.2, where the passively allocated fund holds an exogenous endowment of shares, as would happen if the fund tracked an index. Overall, the effect of passively allocated funds on corporate actions and prices implied by our model

¹²The active-passive dichotomy here relates to stock-picking, rather than to influence or stewardship activities. Passive indexers can and do exert influence activities via voting and engagement.

depends on how such passive funds are interpreted, i.e., whether passive stock picking implies passive ownership without influence or stewardship activities that reflect influence.

5 Conclusion

In this paper, we study the implications of interested investors and intermediaries, where the interests refer to preferences over managerial actions above and beyond their effects on cash flows. We find that, absent intermediaries, such preferences do not affect firms' actions or prices. In our model, intermediaries make portfolio decisions on behalf of their client investors and may have their own preferences over the actions that firms take. When investors delegate their investment decisions, investors' and intermediaries' preferences affect investor activism or influence, managerial actions, stock prices, and portfolio delegation choices. We find that this effect extends beyond just the impact on expected cash flows. This happens because share ownership affects the interest to expend influence efforts, such that preferences affect the marginal value of share holdings. Free-riding on investor activism plays a central role, as influence activities can benefit all shareholders, but their costs are borne privately. In equilibrium, therefore, the existence of delegation requires intermediaries to have some advantage, e.g., via scale (spreading fixed costs) or information asymmetry.

We find that interested intermediaries benefit from preference alignment with managers or with other insiders, because this allows them to economize on privately costly influence efforts, which helps to attract investors' capital. Greater costs to direct investing shift holdings towards intermediaries, which can lead to greater influence efforts and more positive expected firm cash flows while reducing stock price via the effect of costs on aggregate share demand. The effects of changes in influence costs depend on the intermediary's interests and alignment with management. We further demonstrate that investors' nonpecuniary preferences are not sufficient to overcome the free-rider problems of influence and delegation and that intermediaries have incentives to commit to exclude assets that benefit from their influence.

6 Appendix A - Proofs

Proof of Claim in Footnote 8

Let the cost of m_i be a general convex function, $c(m_i)$. The optimal manager action is still $a = m_i + \mu$. For m_i^* , we have the problem:

$$m_{i}^{*} \in \arg\max_{m_{i}} \lambda \left(q_{i} \left(\beta \left(m_{i} + \mu \right) - p \right) - \frac{\gamma}{2} \left(m_{i} + \mu - A_{j} \right)^{2} - \frac{1}{2} \tau q_{i}^{2} \sigma^{2} \right) - \frac{\gamma}{2} \left(m_{i} + \mu - A_{i} \right)^{2} - c \left(m_{i} \right)$$

with first-order condition (FOC):

$$c'(m_i^*) = \lambda q_i \beta - \lambda \gamma \left(m_i^* + \mu - A_j\right) - \gamma \left(m_i^* + \mu - A_i\right).$$
⁽¹⁹⁾

The second-order condition (SOC) is satisfied by sufficient convexity of $c(m_i)$.

Write $m_i^*(q_i)$ to make the dependence of m_i^* on q_i clear. Substituting the intermediary's influence from (19) into (4) yields their expected utility at t = 1, solving the FOC for maximizing the expected utility over the demand of shares:

$$q_{i}^{*} \in \arg \max_{q_{i}} \lambda \left(q_{i} \left(\beta \left(m_{i}^{*} \left(q_{i} \right) + \mu \right) - p \right) - \frac{\gamma}{2} \left(m_{i}^{*} \left(q_{i} \right) + \mu - A_{j} \right)^{2} - \frac{1}{2} \tau q_{i}^{2} \sigma^{2} \right) - \frac{\gamma}{2} \left(m_{i}^{*} \left(q_{i} \right) + \mu - A_{i} \right)^{2} - c \left(m_{i}^{*} \left(q_{i} \right) \right)$$

has FOC with respect to q_i^* of

$$0 = \lambda \left(\beta \left(m_i^* \left(q_i \right) + \mu \right) - \frac{\gamma}{2} \left(m_i^* \left(q_i \right) + \mu - A_j \right)^2 - p \right) + \lambda q_i \beta m_i^{*'} \left(q_i \right) \right. \\ \left. -\lambda \gamma \left(m_i^* \left(q_i \right) + \mu - A_j \right) m_i^{*'} \left(q_i \right) - \lambda \tau q_i \sigma^2 \right. \\ \left. -\gamma \left(m_i^* \left(q_i \right) + \mu - A_i \right) m_i^{*'} \left(q_i \right) - c' \left(m_i^* \left(q_i \right) \right) m_i^{*'} \left(q_i \right) \right.$$
(20)

For the direct investors, demand is given by $\tau \sigma^2 q_j^* = \beta \left(m_i^* \left(q_i^* \right) + \mu \right) - \frac{\gamma}{2} \left(m_i^* \left(q_i \right) + \mu - A_j \right)^2 - p$. We can rearrange equation (20) and substitute using (19) to arrive at the FOC for the delegating investors:

$$0 = \lambda \left(\beta \left(m_{i}^{*} \left(q_{i} \right) + \mu \right) - \frac{\gamma}{2} \left(m_{i}^{*} \left(q_{i} \right) + \mu - A_{j} \right)^{2} - p \right) + \lambda q_{i} \beta m_{i}^{*'} \left(q_{i} \right) \right) \\ -\lambda \gamma \left(m_{i}^{*} \left(q_{i} \right) + \mu - A_{j} \right) m_{i}^{*'} \left(q_{i} \right) - \gamma \left(m_{i}^{*} \left(q_{i} \right) + \mu - A_{i} \right) m_{i}^{*'} \left(q_{i} \right) - \lambda \tau q_{i} \sigma^{2} \\ - \left(\lambda q_{i} \beta - \lambda \gamma \left(m_{i}^{*} + \mu - A_{j} \right) - \gamma \left(m_{i}^{*} + \mu - A_{i} \right) \right) m_{i}^{*'} \left(q_{i} \right) \right) \\ 0 = \lambda \left(\beta \left(m_{i}^{*} \left(q_{i} \right) + \mu \right) - \frac{\gamma}{2} \left(m_{i}^{*} \left(q_{i} \right) + \mu - A_{j} \right)^{2} - p \right) - \lambda \tau q_{i} \sigma^{2} \\ \Leftrightarrow \tau q_{i} \sigma^{2} = \beta \left(m_{i}^{*} \left(q_{i} \right) + \mu - \frac{\gamma}{2} \left(m_{i}^{*} \left(q_{i} \right) + \mu - A_{j} \right)^{2} \right) - p.$$

Note that Admati et al. (1994) generate several results using an 'allocation-neutral' cost function, which effectively is $c(m_i)$. A cost function that is not allocation-neutral would be $c(m_i, q_i)$, with $\frac{dc(m_i, q_i)}{dq_i} \neq 0$.

Proof of Propositions 1, 2, and 3: The optimal influence effort is given by $m_i^*(q_j) \in \arg \max_{m_i} u_i$ as

$$m_i^*(q_i) = \frac{\gamma \left(A_i - \mu\right) + \lambda \gamma \left(A_j - \mu\right) + \beta \lambda q_i}{\left(c + \gamma + \lambda \gamma\right)}.$$
(21)

Substitute this into the expression for u_i in (4), then maximize over q_i to obtain

$$q_i^*\left(p\right) = \frac{\beta\left(c\mu + \gamma A_i + \lambda \gamma A_j\right) - p\left(c + \gamma + \lambda \gamma\right)}{\tau \sigma^2 \left(c + \gamma + \lambda \gamma\right) - \beta^2 \lambda}.$$

Recall that direct investors optimally exert no influence efforts even though they are interested in the manager's action above and beyond its effect on cash flows. Direct investors choose price-contingent demand, $q_j^*(p)$ to maximize CE_j , anticipating how the the shares held by the intermediary will affect the influence effort exerted on the firm. Taking $q_j^*(p) \in$ arg max_{q_j} CE_j ($m_i = m_i^*, q_i = q_i^*$) yields

$$q_j^*(p) = \frac{\beta \left(c\mu + \gamma A_i + \lambda \gamma A_j\right) - p \left(c + \gamma + \lambda \gamma\right)}{\tau \sigma^2 \left(c + \gamma + \lambda \gamma\right) - \beta^2 \lambda},\tag{22}$$

which implies $q_i^*(p) = q_j^*(p)$.

Market clearing defines price as $1 = \lambda q_i^*(p) + (1 - \lambda) q_j^*(p)$, yielding

$$p^* = \beta \frac{c\mu + \gamma A_i + \lambda \gamma A_j + \beta \lambda}{c + \gamma + \lambda \gamma} - \tau \sigma^2.$$
(23)

Substituting the equilibrium price and quantities into the expression for m_i^* above results in the equilibrium influence effort and managerial action of

$$m_i^* = \frac{\beta \lambda + \gamma \left(A_i - \mu\right) + \lambda \gamma \left(A_j - \mu\right)}{c + \gamma + \lambda \gamma}, \text{ and}$$
 (24)

$$a^* = \frac{c\mu + \gamma A_i + \lambda \gamma A_j + \beta \lambda}{c + \gamma + \lambda \gamma}.$$
(25)

The expected utility of a delegating investor can be written as

$$CE_{i}^{*} = \frac{u_{i}^{*}}{\lambda} = CE_{j}^{*} - \left(\frac{1}{2\lambda}\gamma \left(a_{II}^{*} - A_{i}\right)^{2} + c\left(m_{i,II}^{*}\right)^{2}\right).$$
(26)

The equilibrium λ is defined by

$$\kappa = \frac{1}{2\lambda} \left(\gamma \left(a^* - A_i \right)^2 + c \left(m_i^* \right)^2 \right) \Rightarrow$$

$$2\lambda \kappa = \gamma \left(\frac{c\mu + \gamma A_i + \lambda \gamma A_j + \beta \lambda}{c + \gamma + \lambda \gamma} - A_i \right)^2 + c \left(\frac{\beta \lambda + \gamma \left(A_i - \mu \right) + \lambda \gamma \left(A_j - \mu \right)}{c + \gamma + \lambda \gamma} \right)^2 .(27)$$

Rewrite equation (27) as

$$0 = (2\kappa\gamma^{2})\lambda^{3} + (4\kappa\gamma (c+\gamma) - \gamma (\beta + \gamma A_{j} - \gamma A_{i})^{2} - c (\beta - \gamma (\mu - A_{j}))^{2})\lambda^{2} + 2 (\kappa (c+\gamma)^{2} - c\gamma^{2} (\mu - A_{i})^{2})\lambda - c\gamma (\mu - A_{i})^{2} (c+\gamma)$$

and define the right-hand side (RHS) as $G_1(\cdot)$.

$$\frac{\partial G_1}{\partial \lambda} = 6\kappa \gamma^2 \lambda^2 + 2 \left(4\kappa \gamma \left(c + \gamma \right) - \gamma \left(\beta + \gamma A_j - \gamma A_i \right)^2 - c \left(\beta - \gamma \left(\mu - A_j \right) \right)^2 \right) \lambda + 2 \left(\kappa \left(c + \gamma \right)^2 - c \gamma^2 \left(\mu - A_i \right)^2 \right), \text{ and} \frac{\partial G_1}{\partial \kappa} = 2\gamma^2 \lambda^3 + 4\gamma \left(c + \gamma \right) \lambda^2 + 2 \left(c + \gamma \right)^2 \lambda > 0.$$

Note that for any reasonable equilibrium, we should have $\frac{d\lambda}{d\kappa} > 0$.

$$\frac{d\lambda}{d\kappa} = -\frac{\frac{\partial G_1}{\partial \lambda}}{\frac{\partial G_1}{\partial \kappa}} = -\frac{\frac{\partial G_1}{\partial \lambda}}{2\gamma^2 \lambda^3 + 4\gamma \left(c + \gamma\right) \lambda^2 + 2\left(c + \gamma\right)^2 \lambda}$$

which implies $\frac{\partial G_1}{\partial \lambda} < 0$. The partial with respect to the direct investors' preference parameter, A_j , is

$$\frac{\partial G_{1}}{\partial A_{j}} = \frac{\partial}{\partial A_{j}} \begin{pmatrix} (2\kappa\gamma^{2})\lambda^{3} + (4\kappa\gamma(c+\gamma) - \gamma(\beta+\gamma A_{j}-\gamma A_{i})^{2} - c(\beta-\gamma(\mu-A_{j}))^{2})\lambda^{2} \\ + 2(\kappa(c+\gamma)^{2} - c\gamma^{2}(\mu-A_{i})^{2})\lambda - c\gamma(\mu-A_{i})^{2}(c+\gamma) \end{pmatrix} \\
= -2\lambda^{2}\gamma(c\beta+\beta\gamma+c\gamma(A_{j}-\mu)+\gamma^{2}(A_{j}-A_{i}))$$

And, from the chain rule, $\frac{d\lambda}{dA_j} \propto \frac{\partial G_1}{\partial A_j}$. Note that $\frac{\partial G_1}{\partial A_j} \ge 0$. It is positive if

$$A_{j} < \frac{c\gamma\mu + \gamma^{2}A_{i} - c\beta - \beta\gamma}{\gamma(c+\gamma)}$$

So, for low A_j , the fraction of investors who delegate is increasing in A_j .

Proof of Corollary 2:

The derivatives of p with respect to λ , c, and γ are given by

$$\frac{\partial}{\partial\lambda} \left(\beta \frac{c\mu + \gamma A_i + \lambda \gamma A_j + \beta \lambda}{c + (1 + \lambda) \gamma} \right) = \beta \frac{\beta (c + \gamma) + \gamma^2 (A_j - A_i) + c\gamma (A_j - \mu)}{(c + (1 + \lambda) \gamma)^2}, \quad (28)$$

$$\frac{\partial}{\partial c} \left(\beta \frac{c\mu + \gamma A_i + \lambda \gamma A_j + \beta \lambda}{c + (1 + \lambda) \gamma} \right) = -\beta \frac{\beta \lambda + \gamma (A_i - \mu) + \lambda \gamma (A_j - \mu)}{(c + (1 + \lambda) \gamma)^2}, \text{ and} \qquad (29)$$

$$\frac{\partial}{\partial\gamma} \left(\beta \frac{c\mu + \gamma A_i + \lambda \gamma A_j + \beta \lambda}{c + (1 + \lambda) \gamma} \right) = \beta \frac{c (A_i - \mu) + c\lambda (A_j - \mu) - \beta \lambda (1 + \lambda)}{(c + (1 + \lambda) \gamma)^2}.$$
 (30)

Proof of Proposition 4: The relevant derivative is

$$\frac{da^{*}}{dA_{j}} = \frac{d}{dA_{j}} \left(\frac{c\mu + \gamma A_{i} + \lambda \gamma A_{j} + \beta \lambda}{c + \gamma + \lambda \gamma} \right) \\
= \frac{\lambda \gamma}{c + \gamma + \lambda \gamma} + \frac{\beta (c + \gamma) + \gamma (c (A_{j} - \mu) + \gamma (A_{j} - A_{i}))}{(c + \gamma + \lambda \gamma)^{2}} \frac{d\lambda}{dA_{j}}$$
(31)

Proposition 3 implies that $\frac{\beta(c+\gamma)+\gamma(c(A_j-\mu)+\gamma(A_j-A_i))}{(c+\gamma+\lambda\gamma)^2}$ and $\frac{d\lambda}{dA_j}$ have opposite signs, so the indirect effect in the second additive term in equation (31) is negative. Recall that λ^* is defined by $G_1(\cdot) = 0$, where $G_1(\cdot)$ is defined in the proof of Proposition 3 as:

$$G_{1}(\cdot) = (2\kappa\gamma^{2})\lambda^{3} + (4\kappa\gamma(c+\gamma) - \gamma(\beta+\gamma A_{j}-\gamma A_{i})^{2} - c(\beta-\gamma(\mu-A_{j}))^{2})\lambda^{2} + 2(\kappa(c+\gamma)^{2} - c\gamma^{2}(\mu-A_{i})^{2})\lambda - c\gamma(\mu-A_{i})^{2}(c+\gamma).$$

For any reasonable equilibrium, we have

$$\frac{\partial G_1}{\partial \lambda} = 6\kappa \gamma^2 \lambda^2 + 2\left(4\kappa \gamma \left(c+\gamma\right) - \gamma \left(\beta + \gamma A_j - \gamma A_i\right)^2 - c\left(\beta - \gamma \left(\mu - A_j\right)\right)^2\right) \lambda + 2\left(\kappa \left(c+\gamma\right)^2 - c\gamma^2 \left(\mu - A_i\right)^2\right) < 0.$$

From the chain rule, $\frac{d\lambda}{dA_j} = -\frac{\frac{\partial G_1}{\partial \lambda}}{\frac{\partial G_1}{\partial A_j}}$. With

$$\frac{\partial G_1}{\partial A_j} = -2\lambda^2 \gamma \left(c\beta + \beta\gamma + c\gamma \left(A_j - \mu \right) + \gamma^2 \left(A_j - A_i \right) \right)$$

we have

$$\frac{d\lambda}{dA_{j}} = -\frac{\frac{\partial G_{1}}{\partial \lambda}}{-2\lambda^{2}\gamma \left(c\beta + \beta\gamma + c\gamma \left(A_{j} - \mu\right) + \gamma^{2} \left(A_{j} - A_{i}\right)\right)} \qquad (32)$$

$$= -\frac{6\kappa\gamma^{2}\lambda^{2} + 2\left(4\kappa\gamma \left(c + \gamma\right) - \gamma \left(\beta + \gamma A_{j} - \gamma A_{i}\right)^{2} - c\left(\beta - \gamma \left(\mu - A_{j}\right)\right)^{2}\right)\lambda}{-2\lambda^{2}\gamma \left(c\beta + \beta\gamma + c\gamma \left(A_{j} - \mu\right) + \gamma^{2} \left(A_{j} - A_{i}\right)\right)} - \frac{2\left(\kappa \left(c + \gamma\right)^{2} - c\gamma^{2} \left(\mu - A_{i}\right)^{2}\right)}{-2\lambda^{2}\gamma \left(c\beta + \beta\gamma + c\gamma \left(A_{j} - \mu\right) + \gamma^{2} \left(A_{j} - A_{i}\right)\right)}.$$

From (32), we have

$$\frac{da^{*}}{dA_{j}} = \frac{\lambda\gamma}{c+\gamma+\lambda\gamma} + \frac{\left(c\beta+\beta\gamma-\gamma^{2}A_{i}+\gamma^{2}A_{j}-c\gamma\mu+c\gamma A_{j}\right)\left(-\frac{\frac{\partial G_{1}}{\partial\lambda}}{-2\lambda^{2}\gamma(c\beta+\beta\gamma+c\gamma(A_{j}-\mu)+\gamma^{2}(A_{j}-A_{i}))}\right)}{(c+\gamma+\lambda\gamma)^{2}} \\
= \frac{\lambda\gamma}{c+\gamma+\lambda\gamma} + \frac{\frac{\partial G_{1}}{\partial\lambda}}{2\lambda^{2}\gamma(c+\gamma+\lambda\gamma)^{2}} \\
\propto 2\lambda^{3}\gamma^{2}(c+\gamma+\lambda\gamma) + \frac{\partial G_{1}}{\partial\lambda} \\
= 2\lambda^{3}\gamma^{2}(c+\gamma+\lambda\gamma) + 6\kappa\gamma^{2}\lambda^{2} + 2\left(4\kappa\gamma(c+\gamma)-\gamma\left(\beta+\gamma A_{j}-\gamma A_{i}\right)^{2}-c\left(\beta-\gamma\left(\mu-A_{j}\right)\right)^{2}\right)\lambda \\
+ 2\left(\kappa(c+\gamma)^{2}-c\gamma^{2}\left(\mu-A_{i}\right)^{2}\right) \\
= 2(c+\gamma+3\lambda\gamma)(c+\gamma+\lambda\gamma)\kappa+2\lambda^{3}\gamma^{2}(c+\gamma+\lambda\gamma) \qquad (33) \\
-2\lambda\left(c\left(\beta-\gamma\left(\mu-A_{j}\right)\right)^{2}+\gamma\left(\beta-\gamma A_{i}+\gamma A_{j}\right)^{2}\right) - 2c\gamma^{2}\left(\mu-A_{i}\right)^{2}$$

Note that as $\kappa \to 0$, we have $\lambda \to 0$ and taking the line from (33)

$$\begin{split} &\lim_{\kappa \to 0} \left(\begin{array}{c} 2\left(c + \gamma + 3\lambda\gamma\right)\left(c + \gamma + \lambda\gamma\right)\kappa + 2\lambda^{3}\gamma^{2}\left(c + \gamma + \lambda\gamma\right) \\ -2\lambda\left(c\left(\beta - \gamma\left(\mu - A_{j}\right)\right)^{2} + \gamma\left(\beta - \gamma A_{i} + \gamma A_{j}\right)^{2}\right) - 2c\gamma^{2}\left(\mu - A_{i}\right)^{2} \right) \\ &= \lim_{\lambda \to 0} \left(2\lambda^{3}\gamma^{2}\left(c + \gamma + \lambda\gamma\right) - 2\lambda\left(c\left(\beta - \gamma\left(\mu - A_{j}\right)\right)^{2} + \gamma\left(\beta - \gamma A_{i} + \gamma A_{j}\right)^{2}\right) - 2c\gamma^{2}\left(\mu - A_{i}\right)^{2}\right) \\ &= -2c\gamma^{2}\left(\mu - A_{i}\right)^{2} < 0 \end{split}$$

whereas as $\kappa \to \infty$, we have $\lambda \to 1$ and

$$\lim_{\kappa \to \infty} \begin{pmatrix} 2(c+\gamma+3\lambda\gamma)(c+\gamma+\lambda\gamma)\kappa+2\lambda^{3}\gamma^{2}(c+\gamma+\lambda\gamma)\\ -2\lambda\left(c\left(\beta-\gamma\left(\mu-A_{j}\right)\right)^{2}+\gamma\left(\beta-\gamma A_{i}+\gamma A_{j}\right)^{2}\right)-2c\gamma^{2}\left(\mu-A_{i}\right)^{2} \end{pmatrix} \\
= \lim_{\lambda \to 1} \begin{pmatrix} 2(c+\gamma+3\gamma)(c+\gamma+\gamma)\infty+2\gamma^{2}(c+\gamma+\lambda\gamma)\\ -2\left(c\left(\beta-\gamma\left(\mu-A_{j}\right)\right)^{2}+\gamma\left(\beta-\gamma A_{i}+\gamma A_{j}\right)^{2}\right)-2c\gamma^{2}\left(\mu-A_{i}\right)^{2} \end{pmatrix} \\
\propto 2(c+\gamma+3\gamma)(c+\gamma+\gamma) > 0.$$

So $\frac{da^*}{dA_j}$ is negative for low κ and positive for high κ .

Proof of Proposition 5.

When the intermediary trades in both assets, the FOC for the intermediary's influence, as a function of shares held, implies

$$m_{i}^{*}(q_{i1}, q_{i2}) = \frac{\gamma \left(A_{i} - \mu_{1}\right)}{c + \gamma} + \frac{\lambda \left(q_{i1}\beta_{1} + q_{i2}\beta_{2}\right)}{c + \gamma}.$$
(34)

Substituting the intermediary's influence into their expected utility at t = 1 (omitting obvi-

ous subscripts for the moment) yields

$$\begin{aligned} u_i &= \lambda q_1 \left(\beta_1 \left(\frac{\gamma \left(A_i - \mu_1\right)}{c + \gamma} + \frac{\lambda \left(q_1 \beta_1 + q_2 \beta_2\right)}{c + \gamma} + \mu_1 \right) - p_1 \right) \\ &+ \lambda q_2 \left(\beta_2 \left(\frac{\gamma \left(A_i - \mu_1\right)}{c + \gamma} + \frac{\lambda \left(q_1 \beta_1 + q_2 \beta_2\right)}{c + \gamma} + \mu_1 \right) - p_2 \right) \\ &- \lambda \frac{1}{2} \tau \left(q_1^2 \sigma_1^2 + q_2^2 \sigma_2^2 + 2\rho q_1 q_2 \sigma_1 \sigma_2 \right) \\ &- \frac{\gamma}{2} \left(\frac{\gamma \left(A_i - \mu_1\right)}{c + \gamma} + \frac{\lambda \left(q_1 \beta_1 + q_2 \beta_2\right)}{c + \gamma} + \mu_1 - A_i \right)^2 \\ &- \frac{c}{2} \left(\frac{\gamma \left(A_i - \mu_1\right)}{c + \gamma} + \frac{\lambda \left(q_1 \beta_1 + q_2 \beta_2\right)}{c + \gamma} \right)^2. \end{aligned}$$

Solving the set of FOC for maximizing the expected utility over the demands of shares for (q_1, q_2) yields :

$$q_{1} = \frac{\tau \sigma_{2} \frac{\beta_{1} \sigma_{2} - \beta_{2} \rho \sigma_{1}}{c + \gamma} \left(\gamma A_{i} + c \mu_{1}\right) - p_{1} \left(\tau \sigma_{2}^{2} - \frac{\lambda \beta_{2}^{2}}{c + \gamma}\right) + p_{2} \left(\tau \rho \sigma_{1} \sigma_{2} - \frac{\lambda \beta_{1} \beta_{2}}{c + \gamma}\right)}{\left(\tau \sigma_{2}^{2} - \frac{\lambda \beta_{2}^{2}}{c + \gamma}\right) \left(\tau \sigma_{1}^{2} - \frac{\lambda \beta_{1}^{2}}{c + \gamma}\right) - \left(\tau \rho \sigma_{1} \sigma_{2} - \lambda \frac{\beta_{1} \beta_{2}}{c + \gamma}\right)^{2}}\right.}$$

$$q_{2} = \frac{\tau \sigma_{1} \frac{\beta_{2} \sigma_{1} - \beta_{1} \rho \sigma_{2}}{c + \gamma} \left(\gamma A_{i} + c \mu_{1}\right) - p_{2} \left(\tau \sigma_{1}^{2} - \frac{\lambda \beta_{1}^{2}}{c + \gamma}\right) + p_{1} \left(\tau \rho \sigma_{1} \sigma_{2} - \frac{\lambda \beta_{1} \beta_{2}}{c + \gamma}\right)}{\left(\tau \sigma_{2}^{2} - \frac{\lambda \beta_{2}^{2}}{c + \gamma}\right) \left(\tau \sigma_{1}^{2} - \frac{\lambda \beta_{1}^{2}}{c + \gamma}\right) - \left(\tau \rho \sigma_{1} \sigma_{2} - \frac{\lambda \beta_{1} \beta_{2}}{c + \gamma}\right)^{2}}\right.}$$

Substituting demands and rearranging terms allows us to express influence and effort as

$$m_i^* (q_{i1}^*, q_{i2}^*) = \frac{\gamma (A_i - \mu_1)}{c + \gamma} + \frac{\lambda}{c + \gamma} (\beta_1 + \beta_2) \text{ and}$$
$$a_1^* (m_i^*) = \frac{c\mu_1 + \gamma A_i}{c + \gamma} + \frac{\lambda}{c + \gamma} (\beta_1 + \beta_2).$$

A direct investor chooses q_{j1} and q_{j2} to maximize

$$CE_{j} = q_{j1} \left(\beta_{1}a_{1} - p_{1}\right) + q_{j2} \left(\beta_{2}a_{1} - p_{2}\right) - \frac{1}{2}\tau \left(q_{j1}^{2}\sigma_{1}^{2} + q_{j2}^{2}\sigma_{2}^{2} + 2\rho q_{j1}q_{j2}\sigma_{1}\sigma_{2}\right).$$

The set of FOC is given by

$$0 = \beta_1 a_1 - p_1 - \tau \left(q_{j1} \sigma_1^2 + \rho q_{j2} \sigma_1 \sigma_2 \right) \text{ and } 0 = \beta_2 a_1 - p_2 - \tau \left(q_{j2} \sigma_2^2 + \rho q_{j1} \sigma_1 \sigma_2 \right).$$

Solving this for q_{j1} and q_{j2} yields

$$q_{j2} = \frac{\beta_2 a_1 - p_2 - \rho \frac{\sigma_2}{\sigma_1} (\beta_1 a_1 - p_1)}{\tau (1 - \rho^2) \sigma_2^2} = q_{i2} \text{ and}$$
$$q_{j1} = \frac{\beta_1 a_1 - p_1 - \rho \frac{\sigma_1}{\sigma_2} (\beta_2 a_1 - p_2)}{\tau \sigma_1^2 (1 - \rho^2)} = q_{i1}.$$

Market clearing requires

$$1 = \lambda q_{i1} + (1 - \lambda) q_{j1} \text{ and}$$

$$1 = \lambda q_{i2} + (1 - \lambda) q_{j2}.$$

Substituting demands and solving for prices yields the following prices,

$$p_{1} = \beta_{1} \frac{\gamma A_{i} + c\mu_{1} + \lambda \left(\beta_{1} + \beta_{2}\right)}{c + \gamma} - \tau \left(\sigma_{1}^{2} + \rho\sigma_{1}\sigma_{2}\right) \text{ and}$$

$$p_{2} = \beta_{2} \frac{\gamma A_{i} + c\mu_{1} + \lambda \left(\beta_{1} + \beta_{2}\right)}{c + \gamma} - \tau \left(\sigma_{2}^{2} + \rho\sigma_{1}\sigma_{2}\right).$$

The intermediary's and the respective investor's expected utilities are given by

$$u_{i}^{*} = \lambda \frac{1}{2} \tau \left(\sigma_{1}^{2} + \sigma_{2}^{2} + 2\rho \sigma_{1} \sigma_{2} \right) - \frac{1}{2} \frac{\lambda^{2} \left(\beta_{1} + \beta_{2} \right)^{2} + c\gamma \left(A_{i} - \mu_{1} \right)^{2}}{c + \gamma},$$

$$CE_{i}^{*} = \frac{1}{2} \tau \left(\sigma_{1}^{2} + \sigma_{2}^{2} + 2\rho \sigma_{1} \sigma_{2} \right) - \frac{1}{2} \frac{\lambda^{2} \left(\beta_{1} + \beta_{2} \right)^{2} + c\gamma \left(A_{i} - \mu_{1} \right)^{2}}{\lambda \left(c + \gamma \right)}, \text{ and }$$

$$CE_{j} = \frac{1}{2} \tau \left(\sigma_{1}^{2} + \sigma_{2}^{2} + 2\rho \sigma_{1} \sigma_{2} \right).$$

In equilibrium,

$$0 = \lambda^2 \left(\beta_1 + \beta_2\right)^2 - 2\lambda \left(c + \gamma\right) \kappa + c\gamma \left(A_i - \mu_1\right)^2.$$

There are two potential solutions to the equilibrium condition

$$\lambda_{1} = \frac{\kappa (c+\gamma)}{(\beta_{1}+\beta_{2})^{2}} - \sqrt{\left(\frac{\kappa (c+\gamma)}{(\beta_{1}+\beta_{2})^{2}}\right)^{2} - c\gamma \left(\frac{A_{i}-\mu}{\beta_{1}+\beta_{2}}\right)^{2}} \text{ and}$$
$$\lambda_{2} = \frac{\kappa (c+\gamma)}{(\beta_{1}+\beta_{2})^{2}} + \sqrt{\left(\frac{\kappa (c+\gamma)}{(\beta_{1}+\beta_{2})^{2}}\right)^{2} - c\gamma \left(\frac{A_{i}-\mu}{\beta_{1}+\beta_{2}}\right)^{2}}.$$

Note that only in the second solution do we have $\frac{\partial \lambda^*}{\partial \kappa} > 0$. Note also that

$$\frac{\partial \lambda_2}{\partial \beta_2} = -2\kappa \frac{c+\gamma}{\left(\beta_1+\beta_2\right)^3} - \frac{2\frac{\kappa^2 (c+\gamma)^2}{\left(\beta_1+\beta_2\right)^2} - c\gamma \left(A_i-\mu\right)^2}{\left(\beta_1+\beta_2\right)^3 \sqrt{\left(\frac{\kappa (c+\gamma)}{\left(\beta_1+\beta_2\right)^2}\right)^2 - c\gamma \left(\frac{A_i-\mu}{\beta_1+\beta_2}\right)^2}} < 0 \text{ for } (\beta_1+\beta_2) > 0 \text{ and}$$
$$\lim_{\beta_2 \to \infty} \lambda_2 \to 0.$$

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That is, when effort has a positive aggregate impact, the larger the externality, the smaller the fraction of investors that delegate. When the externality grows unbounded, then no investor will delegate.

When the intermediary commits not to trade in the shares of asset 2, the influence effort and demand of the intermediary are then given by the expressions in equations (5) and (6):

$$m_{i}^{*}(q_{i1}) = \frac{\gamma \left(A_{i}-\mu\right)}{c+\gamma} + \frac{\beta_{1}\lambda q_{i1}}{c+\gamma} \text{ and}$$

$$q_{i1} = \frac{\beta_{1}\frac{c\mu+\gamma A_{i}}{c+\gamma} - p_{1}}{\tau\sigma_{1}^{2} - \frac{\beta_{1}^{2}\lambda}{c+\gamma}} \text{ or}$$

$$m_{i}^{*}(q_{i1}^{*}) = \frac{\mu \left(\beta_{1}^{2}\lambda - \tau\gamma\sigma_{1}^{2}\right) - \beta_{1}\lambda p_{1} + A_{i}\tau\gamma\sigma_{1}^{2}}{\tau\sigma_{1}^{2}\left(c+\gamma\right) - \beta_{1}^{2}\lambda}$$

This leads to an effort of

$$a^* = \frac{-\beta_1 \lambda p_1 + A_i \tau \gamma \sigma_1^2 + c \tau \mu \sigma_1^2}{-\beta_1^2 \lambda + \tau \gamma \sigma_1^2 + c \tau \sigma_1^2}$$

The utility of a direct investor is given by the expression in equation (12), such that their demands are given by the solution to the following set of FOC,

$$0 = (\beta_1 a_1 - p_1) - \tau (q_{j1} \sigma_1^2 + \rho q_{j2} \sigma_1 \sigma_2) \text{ and}
0 = (\beta_2 a_1 - p_2) - \tau (q_{j2} \sigma_2^2 + \rho q_{j1} \sigma_1 \sigma_2).$$

Solving this system yields

$$q_{j1} = \frac{a_1 (\beta_1 \sigma_2 - \rho \sigma_1 \beta_2) - \sigma_2 p_1 + \rho \sigma_1 p_2}{\tau \sigma_1^2 \sigma_2 (1 - \rho^2)} \text{ and } q_{2j} = \frac{a_1 (\sigma_1 \beta_2 - \rho \beta_1 \sigma_2) - \sigma_1 p_2 + \rho \sigma_2 p_1}{\tau \sigma_1 \sigma_2^2 (1 - \rho^2)}.$$

Substituting the intermediary's choice of effort yields:

$$q_{j1} = \frac{\frac{-\beta_1 \lambda p_1 + A_i \tau \gamma \sigma_1^2 + c\tau \mu \sigma_1^2}{-\beta_1^2 \lambda + \tau \gamma \sigma_1^2 + c\tau \sigma_1^2} (\beta_1 \sigma_2 - \rho \sigma_1 \beta_2) - \sigma_2 p_1 + \rho \sigma_1 p_2}{\tau \sigma_1^2 \sigma_2 (1 - \rho^2)} \text{ and}$$
$$q_{j2} = \frac{\frac{-\beta_1 \lambda p_1 + A_i \tau \gamma \sigma_1^2 + c\tau \mu \sigma_1^2}{-\beta_1^2 \lambda + \tau \gamma \sigma_1^2 + c\tau \sigma_1^2} (\sigma_1 \beta_2 - \rho \beta_1 \sigma_2) - \sigma_1 p_2 + \rho \sigma_2 p_1}{\tau \sigma_1 \sigma_2^2 (1 - \rho^2)}.$$

The market clearing conditions, $1 = (1 - \lambda) q_{j1} + \lambda q_{i1}$ and $1 = (1 - \lambda) q_{j2}$, imply

$$p_{1} = \beta_{1} \left(\frac{A_{i}\gamma + c\mu}{c + \gamma} + \frac{\lambda\beta_{1}}{c + \gamma} \left(1 + \rho \frac{\sigma_{2}}{\sigma_{1}} \right) \right) - \tau \left(\sigma_{1}^{2} + \rho \sigma_{1} \sigma_{2} \right)$$

$$p_{2} = \beta_{2} \left(\frac{A_{i}\gamma + c\mu}{c + \gamma} + \frac{\lambda\beta_{1}}{c + \gamma} \left(1 + \rho \frac{\sigma_{2}}{\sigma_{1}} \right) \right) - \tau \left(\frac{1 - \lambda\rho^{2}}{1 - \lambda} \sigma_{2}^{2} + \rho \sigma_{1} \sigma_{2} \right)$$

Substituting this back into the demand functions yields the equilibrium demands of

$$q_{i1} = 1 + \rho \frac{\sigma_2}{\sigma_1}$$

$$q_{j1} = 1 - \rho \frac{\sigma_2}{\sigma_1} \frac{\lambda}{1 - \lambda} \text{ and}$$

$$q_{j2} = \frac{1}{1 - \lambda}.$$

The equilibrium monitoring effort and manager's choice are therefore given by

$$m_i^*(q_{i1}^*) = \gamma \frac{A_i - \mu}{c + \gamma} + \lambda \beta_1 \frac{\sigma_1 + \rho \sigma_2}{\sigma_1 (c + \gamma)} \text{ and}$$
$$a^* = \frac{A_i \gamma + c\mu}{c + \gamma} + \lambda \beta_1 \frac{\sigma_1 + \rho \sigma_2}{\sigma_1 (c + \gamma)}.$$

Not that a delegating investor and a direct investor no longer have the same demand for shares. Specifically, $q_{i1} > q_{j1}$ whenever $\rho \frac{\sigma_2}{\sigma_1} > 0$. That is, a direct investor's demand for shares in asset 1 is lower for $\rho > 0$ because a positive correlation of cash flows implies that the risk of the portfolio of an investor who also holds shares in asset 2 increases more than that of an investor who does not hold asset-2 shares.

The certainty-equivalent utility of a direct investor is given by

$$CE_{j} = q_{j1} \left(\beta_{1}a_{1} - p_{1}\right) + q_{j2} \left(\beta_{2}a_{1} - p_{2}\right) - \frac{1}{2}\tau \left(q_{j1}^{2}\sigma_{1}^{2} + q_{j2}^{2}\sigma_{2}^{2} + 2\rho q_{j1}q_{j2}\sigma_{1}\sigma_{2}\right) - \kappa$$
$$= \frac{1}{2}\tau \left(\sigma_{1}^{2} + 2\rho\sigma_{1}\sigma_{2} + \rho^{2}\sigma_{2}^{2} + \sigma_{2}^{2}\frac{1 - \rho^{2}}{\left(1 - \lambda\right)^{2}}\right) - \kappa.$$

The certainty-equivalent utility of a delegating investor is given by

$$CE_{i}^{*} = q_{i1} \left(\beta_{1}a_{1} - p_{1}\right) - \frac{1}{2}\tau q_{i1}^{2}\sigma_{1}^{2} - \frac{\gamma}{2\lambda} \left(a_{1} - A_{i}\right)^{2} - \frac{cm_{i}^{2}}{2\lambda}$$
$$= \frac{1}{2}\tau \left(\sigma_{1} + \rho\sigma_{2}\right)^{2} - \frac{1}{2}\frac{c\gamma\sigma_{1}^{2} \left(A_{i} - \mu\right)^{2} + \lambda^{2}\beta_{1}^{2} \left(\sigma_{1} + \rho\sigma_{2}\right)^{2}}{\lambda\sigma_{1}^{2} \left(c + \gamma\right)}$$

This implies the following equilibrium condition, $F(\lambda) = 0$, where

$$F(\lambda) = \frac{1}{2} \frac{c\gamma \sigma_1^2 (A_i - \mu)^2 + \lambda^2 \beta_1^2 (\sigma_1 + \rho \sigma_2)^2}{\lambda \sigma_1^2 (c + \gamma)} + \frac{1}{2} \tau \sigma_2^2 \frac{1 - \rho^2}{(1 - \lambda)^2} - \kappa$$

First, note that $F(\lambda)$ does not depend on β_2 . As a result, if there exists a $\lambda \in (0, 1]$ such that $F(\lambda) = 0$, then there exists a β_2 such that the fraction of investors that delegate is larger when the intermediary commits not to invest in asset 2. We can rewrite the equilibrium condition

$$F_{2}(\lambda) = c\gamma\sigma_{1}^{2}(A_{i}-\mu)^{2}(1-\lambda)^{2} + \lambda^{2}\beta_{1}^{2}(\sigma_{1}+\rho\sigma_{2})^{2}(1-\lambda)^{2} + \tau\sigma_{2}^{2}(1-\rho^{2})\lambda\sigma_{1}^{2}(c+\gamma) - 2\kappa(1-\lambda)^{2}.$$

That is, the equilibrium condition is a 4^{th} -order polynomial. Note that

$$F_{2}(\lambda = 0) = c\gamma\sigma_{1}^{2}(A_{i} - \mu)^{2} - 2\kappa$$

$$F_{2}(\lambda = 1) = \tau\sigma_{2}^{2}(1 - \rho^{2})\sigma_{1}^{2}(c + \gamma) > 0$$

Thus, for $\kappa > c\gamma \sigma_1^2 (A_i - \mu)^2 - 2\kappa$, $F_2(\lambda = 1) < 0$, such that there exists at least one $\lambda \in (0, 1)$ that solves $F_2(\lambda) = 0$.

Proof of Lemma 2: Understanding that direct investors continue to exert no influence efforts, the manager's action choice is given by

$$a^*(m_i, m_h) \in \arg \max_a - (a - m_i - \mu - m_h)^2$$

$$\Rightarrow a^*(m_i, m_h) = m_i + \mu + m_h.$$

The insider's optimal influence effort is

$$m_{h,IN}^{*} \in \arg \max_{m_{h}} \left(q_{h} \left(\beta a \right) - \frac{1}{2} \tau q_{h}^{2} \sigma^{2} - \frac{\gamma}{2} \left(a - A_{h} \right)^{2} - \frac{c}{2} m_{h}^{2} \right) \\ = \arg \max_{m_{h}} \left(q_{h} \left(\beta \left(m_{i} + \mu + m_{h} \right) \right) - \frac{1}{2} \tau q_{h}^{2} \sigma^{2} - \frac{\gamma}{2} \left(m_{i} + \mu + m_{h} - A_{h} \right)^{2} - \frac{c}{2} m_{h}^{2} \right)$$

with FOC

$$-(cm_h + \gamma\mu + \gamma m_h + \gamma m_i - \beta q_h - \gamma A_h) = 0$$

implying

$$m_{h,IN}^{*}\left(q_{h},m_{i}\right) = \frac{\beta q_{h} + \gamma \left(A_{h} - \mu - m_{i}\right)}{c + \gamma}$$

The intermediary's optimal influence effort is

$$m_{i,IN}^{*} \in \arg \max_{m_{i}} \lambda \left(q_{i} \left(\beta a\right) - \frac{1}{2} \tau q_{i}^{2} \sigma^{2} \right) - \frac{\gamma}{2} \left(a - A_{i}\right)^{2} - \frac{c}{2} m_{i}^{2}$$

$$= \arg \max_{m_{i}} \lambda \left(q_{i} \left(\beta \left(m_{i} + \mu + m_{h}\right)\right) - \frac{1}{2} \tau q_{i}^{2} \sigma^{2} \right) - \frac{\gamma}{2} \left(m_{i} + \mu + m_{h} - A_{i}\right)^{2} - \frac{c}{2} m_{i}^{2}$$

with FOC

$$-(cm_i + \gamma\mu + \gamma m_h + \gamma m_i - \gamma A_i - \beta \lambda q_i) = 0$$

implying

$$m_{i,IN}^{*}\left(q_{i},m_{h}\right) = \frac{\beta\lambda q_{i} + \gamma\left(A_{i} - \mu - m_{h}\right)}{c + \gamma}$$

Note that the optimal influence efforts from the intermediary and insider depend on each

other. We solve for the Nash equilibrium in the subgame:

$$m_{h,IN}^{*}(q_{i},q_{h}) = \frac{\gamma^{2}(A_{h}-A_{i})+c\gamma(A_{h}-\mu)+\beta(q_{h}(c+\gamma)-\lambda\gamma q_{i})}{2c\gamma+c^{2}}, \text{ and}$$
$$m_{i,IN}^{*}(q_{i},m_{h}) = \frac{\gamma^{2}(A_{i}-A_{h})+c\gamma(A_{i}-\mu)+\beta(\lambda q_{i}(c+\gamma)-\gamma q_{h})}{c(c+2\gamma)}.$$

Proof of Proposition 6: The intermediary's chooses demand as

$$\begin{aligned} q_{i,IN}^{*}\left(p,q_{h}\right) &\in \arg\max_{q_{i}}\lambda\left(q_{i}\left(\beta\frac{c\mu+\gamma A_{h}+\gamma A_{i}+\beta q_{h}+\beta\lambda q_{i}}{c+2\gamma}-p\right)-\frac{1}{2}\tau q_{i}^{2}\sigma^{2}\right) \\ &\quad -\frac{\gamma}{2}\left(\frac{c\mu+\gamma A_{h}+\gamma A_{i}+\beta q_{h}+\beta\lambda q_{i}}{c+2\gamma}-A_{i}\right)^{2} \\ &\quad -\frac{c}{2}\left(\frac{\gamma^{2}\left(A_{i}-A_{h}\right)+c\gamma\left(A_{i}-\mu\right)+\beta\left(\lambda q_{i}\left(c+\gamma\right)-\gamma q_{h}\right)\right)}{c\left(c+2\gamma\right)}\right)^{2} \\ &= \arg\max_{q_{i}}-\frac{1}{2}\lambda\frac{-\beta^{2}\lambda\left(c\gamma-\gamma^{2}+c^{2}\right)+c\tau\sigma^{2}\left(c+2\gamma\right)^{2}}{c\left(c+2\gamma\right)^{2}}q_{i}^{2} \\ &\quad +\lambda\frac{-c\left(c+2\gamma\right)^{2}p+\beta\left(\left(c+\gamma\right)^{2}\left(c\mu+\beta q_{h}+\gamma A_{h}\right)+\gamma\left(c\gamma-\gamma^{2}+c^{2}\right)A_{i}\right)}{c\left(c+2\gamma\right)^{2}} \\ &\quad -\frac{1}{2}\gamma\left(c+\gamma\right)\frac{\left(c\mu-cA_{i}+\beta q_{h}+\gamma A_{h}-\gamma A_{i}\right)^{2}}{c\left(c+2\gamma\right)^{2}} \end{aligned}$$

which has FOC:

$$0 = -\lambda \frac{-\beta^2 \lambda (c\gamma - \gamma^2 + c^2) + c\tau \sigma^2 (c + 2\gamma)^2}{c (c + 2\gamma)^2} q_i + \lambda \frac{-c (c + 2\gamma)^2 p + \beta ((c + \gamma)^2 (c\mu + \beta q_h + \gamma A_h) + \gamma (c\gamma - \gamma^2 + c^2) A_i)}{c (c + 2\gamma)^2}$$

implying

$$q_{i,IN}^{*}(p,q_{h}) = \frac{\beta \left((c+\gamma)^{2} (c\mu + \beta q_{h} + \gamma A_{h}) + \gamma (c\gamma - \gamma^{2} + c^{2}) A_{i} \right) - c (c+2\gamma)^{2} p}{c\tau\sigma^{2} (c+2\gamma)^{2} - \beta^{2}\lambda (c\gamma - \gamma^{2} + c^{2})}$$
$$\lim_{A_{h} \to \mu, q_{h} \to 0} q_{i,IN}^{*} = \frac{\beta \left((c+\gamma)^{2} (c\mu + \gamma\mu) + \gamma (c\gamma - \gamma^{2} + c^{2}) A_{i} \right) - c (c+2\gamma)^{2} p}{c\tau\sigma^{2} (c+2\gamma)^{2} - \beta^{2}\lambda (c\gamma - \gamma^{2} + c^{2})}$$

The quantity demanded by the direct investors is

$$q_{j,IN}^{*}(p) \in \arg\max_{q_{j}} q_{j} \left(\frac{\beta}{c+2\gamma} \left(\frac{c\mu + \gamma A_{h} + \gamma A_{i} + \beta q_{h}}{\frac{\beta\left((c+\gamma)^{2}(c\mu+\beta q_{h}+\gamma A_{h})+\gamma\left(c\gamma-\gamma^{2}+c^{2}\right)A_{i}\right)-c(c+2\gamma)^{2}p}{c\tau\sigma^{2}(c+2\gamma)^{2}-\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2})}} \right) \right) - p \right)$$
$$-\frac{1}{2}\tau q_{j}^{2}\sigma^{2}$$

which has FOC

$$0 = -\tau\sigma^2 q_j + \left(\frac{\beta}{c+2\gamma} \left(\begin{array}{c} c\mu + \beta q_h + \gamma A_h + \gamma A_i \\ -\beta\lambda \frac{\beta\left((c+\gamma)^2(c\mu+\beta q_h+\gamma A_h)+\gamma\left(c^2+c\gamma-\gamma^2\right)A_i\right)-cp(c+2\gamma)^2}{\beta^2\lambda(c^2+c\gamma-\gamma^2)-c\tau\sigma^2(c+2\gamma)^2}}\end{array}\right) - p\right)$$

that implies

$$q_{j,IN}^{*}(p) = \frac{\beta \left(\left(\beta^{2} \lambda \gamma + c^{2} \tau \sigma^{2} + 2 c \tau \gamma \sigma^{2} \right) \left(c \mu + \beta q_{h} + \gamma A_{h} \right) + \left(\tau c^{2} \gamma \sigma^{2} + 2 \tau c \gamma^{2} \sigma^{2} \right) A_{i} \right)}{\tau \sigma^{2} \left(c \tau \sigma^{2} \left(c + 2 \gamma \right)^{2} - \beta^{2} \lambda \left(c \gamma - \gamma^{2} + c^{2} \right) \right)} - \frac{\left(\beta^{2} \lambda \gamma \left(c + \gamma \right) + c \tau \sigma^{2} \left(c + 2 \gamma \right)^{2} \right) p}{\tau \sigma^{2} \left(c \tau \sigma^{2} \left(c + 2 \gamma \right)^{2} - \beta^{2} \lambda \left(c \gamma - \gamma^{2} + c^{2} \right) \right)}.$$

The price is thus given by

$$1 - q_{h} = \lambda q_{i,IN}^{*} + (1 - \lambda) q_{j,IN}^{*}$$

$$1 - q_{h} = \lambda \frac{\beta \left((c + \gamma)^{2} \left(c\mu + \beta q_{h} + \gamma A_{h} \right) + \gamma \left(c\gamma - \gamma^{2} + c^{2} \right) A_{i} \right) - c \left(c + 2\gamma \right)^{2} p}{c\tau \sigma^{2} \left(c + 2\gamma \right)^{2} - \beta^{2} \lambda \left(c\gamma - \gamma^{2} + c^{2} \right)}$$

$$+ (1 - \lambda) \frac{- \left(\beta^{2} \lambda \gamma \left(c + \gamma \right) + c\tau \sigma^{2} \left(c + 2\gamma \right)^{2} \right) p}{\tau \sigma^{2} \left(c\tau \sigma^{2} \left(c + 2\gamma \right)^{2} - \beta^{2} \lambda \left(c\gamma - \gamma^{2} + c^{2} \right) \right)}$$

$$+ (1 - \lambda) \frac{\beta \left(\left(\beta^{2} \lambda \gamma + c^{2} \tau \sigma^{2} + 2c\tau \gamma \sigma^{2} \right) \left(c\mu + \beta q_{h} + \gamma A_{h} \right) + \left(\tau c^{2} \gamma \sigma^{2} + 2\tau c\gamma^{2} \sigma^{2} \right) A_{i} \right)}{\tau \sigma^{2} \left(c\tau \sigma^{2} \left(c + 2\gamma \right)^{2} - \beta^{2} \lambda \left(c\gamma - \gamma^{2} + c^{2} \right) \right)}$$

which implies

$$p = -\frac{q_{h} - \beta\lambda \frac{(c+\gamma)^{2}(c\mu+\beta q_{h}+\gamma A_{h}) + \gamma \left(c\gamma-\gamma^{2}+c^{2}\right)A_{i}}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2}) - c\tau\sigma^{2}(c+2\gamma)^{2}} - \frac{1}{\tau\sigma^{2}} \frac{(c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma))(\lambda-1)}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2}) - c\tau\sigma^{2}(c+2\gamma)^{2}}}{\frac{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2}) - c\tau\sigma^{2}(c+2\gamma)^{2}}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2}) - c\tau\sigma^{2}(c+2\gamma)^{2}}}} - \frac{\frac{\beta}{\tau\sigma^{2}} \frac{(\lambda-1)\left((c\mu+\beta q_{h}+\gamma A_{h})\left(\beta^{2}\lambda\gamma+c^{2}\tau\sigma^{2}+2c\tau\gamma\sigma^{2}\right)+\left(2c\tau\gamma^{2}\sigma^{2}+c^{2}\tau\gamma\sigma^{2}\right)A_{i}\right)}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2}) - c\tau\sigma^{2}(c+2\gamma)^{2}}}}{\frac{c\lambda(c+2\gamma)^{2}}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2}) - c\tau\sigma^{2}(c+2\gamma)^{2}}} - \frac{1}{\tau\sigma^{2}} \frac{(c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma))(\lambda-1)}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2}) - c\tau\sigma^{2}(c+2\gamma)^{2}}}}{\frac{c\lambda(c+2\gamma)^{2}}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2}) - c\tau\sigma^{2}(c+2\gamma)^{2}}} - \frac{1}{\tau\sigma^{2}} \frac{(c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma))(\lambda-1)}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2}) - c\tau\sigma^{2}(c+2\gamma)^{2}}}}{\frac{c\lambda(c+2\gamma)^{2}}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2}) - c\tau\sigma^{2}(c+2\gamma)^{2}}} + \frac{1}{\tau\sigma^{2}} \frac{c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma)}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2}) - c\tau\sigma^{2}(c+2\gamma)}} + \frac{1}{\tau\sigma^{2}} \frac{(c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma))(\lambda-1)}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2}) - c\tau\sigma^{2}(c+2\gamma)^{2}}} + \frac{1}{\tau\sigma^{2}} \frac{c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma)}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2}) - c\tau\sigma^{2}(c+2\gamma)}} + \frac{1}{\tau\sigma^{2}} \frac{(c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma))(\lambda-1)}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2}) - c\tau\sigma^{2}(c+2\gamma)} + \frac{1}{\tau\sigma^{2}} \frac{c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma)}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2})} + \frac{1}{\tau\sigma^{2}} \frac{(c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma))(\lambda-1)}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2})} + \frac{1}{\tau\sigma^{2}} \frac{(c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma)}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2})} + \frac{1}{\tau\sigma^{2}} \frac{(c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma))(\lambda-1)}{\beta^{2}\lambda(c\gamma-\gamma^{2}+c^{2})} + \frac{1}{\tau\sigma^{2}} \frac{(c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma)}{\gamma^{2}} + \frac{1}{\tau\sigma^{2}} \frac{(c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma)}{\gamma^{2}} + \frac{1}{\tau\sigma^{2}} \frac{(c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma))}{\gamma^{2}} + \frac{1}{\tau\sigma^{2}} \frac{(c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma))}{\gamma^{2}} + \frac{1}{\tau\sigma^{2}} + \frac{1}{\tau\sigma^{2}} \frac{(c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma)}{\gamma^{2}} + \frac{1}{\tau\sigma^{2}} + \frac{1}{\tau\sigma^{2}} \frac{(c\tau\sigma^{2}(c+2\gamma)^{2}+\beta^{2}\lambda\gamma(c+\gamma))}{\gamma^{2}} + \frac{1}{\tau\sigma^{2}} + \frac{1}{\tau\sigma^{2$$

Plugging p_{IN}^{*} into the expressions for $q_{i,IN}^{*}(p,q_{h})$ and $q_{j,IN}^{*}(p,q_{h})$ yields

$$q_{i,IN}^{*}(q_{h}) = \frac{\beta\gamma^{2}(1-\lambda)\left(c\left(\mu-A_{i}\right)+\gamma\left(A_{h}-A_{i}\right)\right)+c\tau\sigma^{2}\left(c+2\gamma\right)^{2}}{c\tau\sigma^{2}\left(c+2\gamma\right)^{2}+\beta^{2}\lambda\gamma\left(1-\lambda\right)\left(c+\gamma\right)} + \frac{\left(\beta^{2}\gamma^{2}\left(1-\lambda\right)-c\tau\sigma^{2}\left(c+2\gamma\right)^{2}\right)q_{h}}{c\tau\sigma^{2}\left(c+2\gamma\right)^{2}+\beta^{2}\lambda\gamma\left(1-\lambda\right)\left(c+\gamma\right)}, \text{ and }$$

$$q_{j,IN}^{*}(q_{h}) = \frac{\beta\lambda\gamma^{2}\left(c\left(A_{i}-\mu\right)+\gamma\left(A_{i}-A_{h}\right)\right)+c\tau\sigma^{2}\left(c+2\gamma\right)^{2}+\beta^{2}\lambda\gamma\left(c+\gamma\right)}{c\tau\sigma^{2}\left(c+2\gamma\right)^{2}+\beta^{2}\lambda\gamma\left(1-\lambda\right)\left(c+\gamma\right)} - \frac{-\left(c+2\gamma\right)\left(\beta^{2}\lambda\gamma+c\tau\sigma^{2}\left(c+2\gamma\right)\right)q_{h}}{c\tau\sigma^{2}\left(c+2\gamma\right)^{2}+\beta^{2}\lambda\gamma\left(1-\lambda\right)\left(c+\gamma\right)}.$$

The Proposition follows from differentiating equilibrium amounts with respect to A_h .

Proof of Proposition 7: The Proposition follows from differentiating equilibrium amounts shown in the text and the proof of Proposition 6 with respect to q_h . For the two bulleted points, we have

$$\frac{dq_{i,IN}^*}{dq_h} \propto \beta^2 \gamma^2 \left(1 - \lambda\right) - c\tau \sigma^2 \left(c + 2\gamma\right)^2$$

and

$$\frac{da_{IN}^*}{dq_h} = \frac{\beta}{c+2\gamma} + \frac{\beta\lambda}{c+2\gamma} \left(\frac{dq_{i,IN}^*}{dq_h}\right).$$
(35)

We can view $\frac{\beta}{c+2\gamma}$ as the direct effect and $\frac{\beta\lambda}{c+2\gamma} \left(\frac{dq_{i,IN}^*}{dq_h}\right)$ as the indirect effect that operates through the delegating investors' holdings. The direct effect is positive. From Proposition 6, the indirect effect is positive if $\beta^2\gamma^2(1-\lambda) > c\tau\sigma^2(c+2\gamma)^2 \iff \lambda < \frac{\beta^2\gamma^2 - c\tau\sigma^2(c+2\gamma)^2}{\beta^2\gamma^2}$, so

this also implies that the total effect is positive. For λ sufficiently high and $c + 2\gamma < 1$, the negative indirect effect can dominate the direct effect, leading to a negative overall effect of insider holdings on influence and managerial efforts.

$$q_{i,IN}^{*}(q_{h}) = \frac{\beta\gamma^{2}(1-\lambda)\left(c\left(\mu-A_{i}\right)+\gamma\left(A_{h}-A_{i}\right)\right)+c\tau\sigma^{2}\left(c+2\gamma\right)^{2}}{c\tau\sigma^{2}\left(c+2\gamma\right)^{2}+\beta^{2}\lambda\gamma\left(1-\lambda\right)\left(c+\gamma\right)} + \frac{\left(\beta^{2}\gamma^{2}\left(1-\lambda\right)-c\tau\sigma^{2}\left(c+2\gamma\right)^{2}\right)q_{h}}{c\tau\sigma^{2}\left(c+2\gamma\right)^{2}+\beta^{2}\lambda\gamma\left(1-\lambda\right)\left(c+\gamma\right)}, \text{ so} \\ \frac{dq_{i,IN}^{*}}{dq_{h}} = \frac{\left(\beta^{2}\gamma^{2}\left(1-\lambda\right)-c\tau\sigma^{2}\left(c+2\gamma\right)^{2}\right)}{c\tau\sigma^{2}\left(c+2\gamma\right)^{2}+\beta^{2}\lambda\gamma\left(1-\lambda\right)\left(c+\gamma\right)}.$$
(36)

Substituting (36) into (35) yields

$$\begin{aligned} \frac{da_{IN}^*}{dq_h} &= \frac{\beta}{c+2\gamma} + \frac{\beta\lambda}{c+2\gamma} \left(\frac{\left(\beta^2\gamma^2\left(1-\lambda\right) - c\tau\sigma^2\left(c+2\gamma\right)^2\right)}{c\tau\sigma^2\left(c+2\gamma\right)^2 + \beta^2\lambda\gamma\left(1-\lambda\right)\left(c+\gamma\right)} \right) \\ &= \frac{\beta}{c+2\gamma} \left(1 + \frac{\lambda}{\left(c+2\gamma\right)} \left(\frac{\left(\beta^2\gamma^2\left(1-\lambda\right) - c\tau\sigma^2\left(c+2\gamma\right)^2\right)}{\left(c\tau\sigma^2\left(c+2\gamma\right)^2 + \beta^2\lambda\gamma\left(1-\lambda\right)\left(c+\gamma\right)\right)} \right) \right) \\ &\propto (c+2\gamma) c\tau\sigma^2\left(c+2\gamma\right)^2 + (c+2\gamma) \beta^2\lambda\gamma\left(1-\lambda\right)\left(c+\gamma\right) + \lambda\beta^2\gamma^2\left(1-\lambda\right) - \lambda c\tau\sigma^2\left(c+2\gamma\right)^2 \\ &= c\sigma^2\tau\left(c+2\gamma\right)^2\left(c+2\gamma-\lambda\right) + \beta^2\lambda\gamma\left(1-\lambda\right)\left(\gamma\left(c+\gamma+1\right) + \left(c+\gamma\right)^2\right) \end{aligned}$$

If $(c + 2\gamma - \lambda) > 0$, then $\frac{da_{IN}^*}{dq_h} > 0 \Leftrightarrow \tau c\sigma^2 > -\frac{\beta^2 \lambda \gamma (\gamma + 3c\gamma + 2\gamma^2 + c^2)(1-\lambda)}{(c+2\gamma)^2 (c+2\gamma-\lambda)}$, which is true. If $(c + 2\gamma - \lambda) < 0$, then $\frac{da_{IN}^*}{dq_h} > 0 \Leftrightarrow$

$$\tau c\sigma^{2} (c+2\gamma)^{2} (c+2\gamma-\lambda) > -\beta^{2} \lambda \gamma \left(\gamma + 3c\gamma + 2\gamma^{2} + c^{2}\right) (1-\lambda)$$

$$\Leftrightarrow \tau c\sigma^{2} < \frac{\beta^{2} \lambda \gamma \left(\gamma + 3c\gamma + 2\gamma^{2} + c^{2}\right) (1-\lambda)}{(c+2\gamma)^{2} (\lambda - c - 2\gamma)}$$

For $\frac{da_{IN}^*}{dq_h} < 0$, we therefore require $c + 2\gamma < \lambda$ and $\frac{\tau\sigma^2}{\beta^2\lambda(1-\lambda)} > \frac{\gamma(\gamma+3c\gamma+2\gamma^2+c^2)}{c(c+2\gamma)^2(\lambda-c-2\gamma)}$.

Appendix B - Alternative settings

Intermediary has private cash flow information

In this extension, we introduce private information available to the intermediary prior to trading. Specifically, we introduce a random shock to the intermediary's cost of influence, such that the cost is $\frac{c}{2}m_i^2 - ym_i$, with $y \sim N(0, \sigma_y^2)$. Marinovic and Varas (2019) use a similar

functional form to introduce uncertainty. They refer to uncertainty about the intermediary's ability or preferences. For our purposes, uncertainty about the cost of influence efforts lead to variation in influence efforts and, through the effect on the manager's efforts, cash flows. That is, variation in the intermediary's cost of effort serves as a convenient means of introducing private information about the firm's cash flows.

Similar to our other extensions, we assume that investors do not have non-monetary preferences, this does not affect the qualitative results in this extension. We assume that the intermediary observes y prior to the opening of the stock market. The intermediary's demand quantity is therefore dependent on the realization of y, while direct investors choose share demand based on their expectation of the intermediary's efficacy, y.

Solving via backward induction, as in the main model, we have $a = m_i + \mu$. The optimal influence effort conditional on shares held is

$$m_{i,PI}^{*}(q_{i},y) \in \arg\max_{m_{i}} \lambda\left(q_{i}\left(\beta a - p\right) - \frac{1}{2}\tau q_{i}^{2}\sigma^{2}\right) - \frac{\gamma}{2}\left(a - A_{i}\right)^{2} - \frac{cm_{i}^{2}}{2} + ym_{i}$$

which implies

$$m_{i,PI}^{*}\left(q_{i}\right) = \frac{y + \gamma\left(A_{i} - \mu\right) + \beta\lambda q_{i}}{c + \gamma}$$

Plugging this into the intermediary's objective and maximizing over q_i yields the optimal quantity of shares demanded as a function of the stock price, p, and the intermediary's efficacy shock, y:

$$q_{i,PI}^{*}(y,p) = \frac{\beta \left(y + c\mu + \gamma A_{i}\right) - p\left(c + \gamma\right)}{\tau \sigma^{2} \left(c + \gamma\right) - \beta^{2} \lambda}.$$

From a direct investor's perspective, the firm will have cash flows of

$$\beta a + \varepsilon = \beta m_i + \beta \mu$$

= $\beta \frac{y\tau\sigma^2 + \tau\sigma^2(c\mu + \gamma A_i) - \beta\lambda p}{\tau\sigma^2(c+\gamma) - \beta^2\lambda}$

which implies cash flows are normally distributed as:

$$x \sim N\left(\beta \frac{\tau \sigma^2 \left(c\mu + \gamma A_i\right) - \beta \lambda p}{\tau \sigma^2 \left(c + \gamma\right) - \beta^2 \lambda}, \left(\frac{\beta \tau \sigma^2}{\tau \sigma^2 \left(c + \gamma\right) - \beta^2 \lambda}\right)^2 \sigma_y^2 + \sigma^2\right).$$

The random shock to the intermediary's efficacy makes cash flows more random from any direct investor's perspective.

For simplicity, we assume that direct investors do not infer y from q_i , which could in turn be inferred from q_j . Introducing noisy supply (i.e.,. the shares available for trade are random rather than 1) would provide a mechanism for limiting investors' ability to infer yfrom market-clearing. However, our focus is on how the intermediary's private information affects quantities held and ex ante utilities of delegating and direct investors. Allowing direct investors to learn from price would reduce and, in the limit eliminate, the intermediary's information advantage.

Direct investors' share demand is thus $q_j = \frac{E[x]-p}{\tau \sigma_x^2}$, or

$$q_{j,PI}^{*}\left(p\right) = \frac{\left(\beta\left(c\mu + \gamma A_{i}\right) - p\left(c + \gamma\right)\right)\left(\tau\sigma^{2}\left(c + \gamma\right) - \beta^{2}\lambda\right)^{2}}{\left(\tau\sigma^{2}\left(c + \gamma\right) - \beta^{2}\lambda\right)\left(\left(\tau\sigma^{2}\left(c + \gamma\right) - \beta^{2}\lambda\right)^{2} + \beta^{2}\tau^{2}\sigma^{2}\sigma_{y}^{2}\right)}$$

Market clearing for a given y implies $1 = \lambda q_i + (1 - \lambda) q_j$. Substituting $q_{i,PI}^*(p)$ and $q_{j,PI}^*(p)$ from above and rearranging yields the equilibrium price. Substituting this price into $q_{i,PI}^*(p)$ and $q_{j,PI}^*(p)$ yield equilibrium quantities.

Proposition 8 When the intermediary has pre-trade private information about its efficacy, given by y:

1. the market-clearing price is

$$p_{PI}^{*} = \frac{\begin{pmatrix} \beta\lambda\left(\beta^{4}\lambda^{2} - \beta^{2}\tau\sigma^{2}\left(2c\lambda + 2\lambda\gamma - \tau\sigma_{y}^{2}\right) + \tau^{2}\sigma^{4}\left(c+\gamma\right)^{2}\right)y \\ + \left(c\beta^{5}\lambda^{2} - c\beta^{3}\lambda\tau\left(2c + 2\gamma - \tau\sigma_{y}^{2}\right)\sigma^{2} + c\beta\tau^{2}\left(c+\gamma\right)^{2}\sigma^{4}\right)\mu \\ + \left(\beta^{5}\lambda^{2}\gamma - \beta^{3}\lambda\tau\gamma\left(2c + 2\gamma - \tau\sigma_{y}^{2}\right)\sigma^{2} + \beta\tau^{2}\gamma\left(c+\gamma\right)^{2}\sigma^{4}\right)A_{i} \\ - \left(\beta^{4}\lambda^{2} - \beta^{2}\tau\sigma^{2}\left(2c\lambda + 2\lambda\gamma - \tau\sigma_{y}^{2}\right) + \tau^{2}\sigma^{4}\left(c+\gamma\right)^{2}\right)\left(\tau\sigma^{2}\left(c+\gamma\right) - \beta^{2}\lambda\right) \end{pmatrix}}{(c+\gamma)\left(\beta^{4}\lambda^{2} + \tau^{2}\sigma^{4}\left(c+\gamma\right)^{2} - \beta^{2}\lambda\tau\sigma^{2}\left(2c + 2\gamma - \tau\sigma_{y}^{2}\right)\right)},$$

2. quantity held by delegating investors is

$$q_{i,PI}^{*} = \frac{\beta \left(1 - \lambda\right) \left(\tau \sigma^{2} \left(c + \gamma\right) - \beta^{2} \lambda\right) y + \left(\tau \sigma^{2} \left(c + \gamma\right) - \beta^{2} \lambda\right)^{2} + \beta^{2} \tau^{2} \sigma^{2} \sigma_{y}^{2}}{\left(\tau \sigma^{2} \left(c + \gamma\right) - \beta^{2} \lambda\right)^{2} + \beta^{2} \lambda \tau^{2} \sigma^{2} \sigma_{y}^{2}}$$

3. and quantity held by direct investors is

$$q_{j,PI}^{*} = \frac{\left(\tau\sigma^{2}\left(c+\gamma\right) - \beta^{2}\lambda\right)^{2} - y\beta\lambda\left(\tau\sigma^{2}\left(c+\gamma\right) - \beta^{2}\lambda\right)}{\left(\tau\sigma^{2}\left(c+\gamma\right) - \beta^{2}\lambda\right)^{2} + \beta^{2}\lambda\tau^{2}\sigma^{2}\sigma_{y}^{2}}.$$

Proposition 8 provides share prices and quantities held in equilibrium as functions of y. The shares held by delegating (direct) investors are increasing (decreasing) in the intermediary's influence efficacy, y. Higher realizations of y lower the intermediary's marginal cost of influence, causing it to demand more shares and leaving fewer shares to direct investors. Even for y = 0, though, delegating investors will tend to hold more shares than direct investors. This occurs because direct investors bear additional risk from the intermediary's efficacy shock. Delegating investors have delegated the quantity choice to the intermediary, who knows y when choosing the demand quantity.

The extension with an intermediary endowed with private cash-flow relevant information parallels Grossman and Stiglitz (1980). In their model, investors choose whether to acquire a costly signal about the firm's cash flows. Here, if we endogenize λ , investors would choose whether or not to delegate their portfolio choice to the intermediary. Although investors do not observe the intermediary's private information directly, they benefit from it because it is used to inform the intermediary's portfolio choice made on behalf of delegating investors. While Grossman and Stiglitz (1980) assumed an exogenous cost of information, the cost of making informed trading decisions in our model comes from the costly actions the intermediary will take in equilibrium. Although these costly actions can benefit cash flows, that benefit accrues to all investors, whether delegating or not. As highlighted earlier, it is only the delegating investors who bear the cost of the influence efforts.

Corollary 4 When the intermediary has pre-trade private information about its efficacy, given by y, an increase in the variance of the intermediary's efficacy, σ_y^2 , leads to a decrease in the expected stock price, $E[p_{PI}^*]$, an increase in the intermediary's expected share holdings, $E[q_{i,PI}^*]$, a decrease in direct investors' expected holdings, $E[q_{j,PI}^*]$, and increases in the expected influence effort, $E[m_{i,PI}^*]$, managerial action, $E[a_{PI}^*]$, and cash flows, $E[\beta a_{PI}^*]$.

Increasing σ_y^2 means that the intermediary's privately-observed efficacy has higher variance, increasing the risk imposed on direct investors. Direct investors react by reducing demand. Although delegating investors increase their demand in expectation, the net effect is to lower the price at which the market for the firm's shares clears. The effects on influence effort, managerial action, and cash flows are direct results of these increasing in the intermediary's shareholdings. Interestingly, when considering the total effect on stock price, the increase in expected cash flows does not offset the decrease in direct investor demand.

Direct investors make trading errors

To introduce an information advantage for the intermediary (without making the monitoring action uncertain from the investors' perspective) we next assume that any investor who invests directly invests based on incorrect beliefs about ε , the noise in cash flows, driven by a signal, y_j , observed by each direct investor. Direct investors believe $y_j = \varepsilon + \varepsilon_j$ with each ε_j independently and identically distributed as $\varepsilon_j \sim N(0, \sigma_j^2)$. In reality, y_j is pure noise, i.e., $y_j = \varepsilon_j \sim N(0, \sigma_j^2)$. The intermediary knows that the y_i are uninformative and ignores them. Direct investors know ex ante that they will react incorrectly to noisy signals, but cannot in the moment stop themselves. In some sense, they cannot tell noisy signals from informative signals, though we do not explicitly model informative signals in this subsection. See Bushee and Friedman (2016) for a model of mood-susceptible investors that is similar in spirit. The noise trader model of De Long et al. (1990) is also similar. The belief that noise represents true information captures investors overconfidence, in that the direct investors overvalue their private signals.

For the period in which investors choose whether to delegate their investment choices to the intermediary or invest directly, they anticipate receipt of the signal and their reaction to it. This means that by investing through the intermediary, they avoid an irrational investment but incur the costs of influence efforts (via a transfer to the intermediary to cover her costs). This has a flavor similar to Grossman and Stiglitz (1980), but with direct investors avoiding mistakes rather than obtaining a costly and truly informative signal on which to base rational decisions.

The intermediary's optimal influence efforts and share demand are unchanged from the main model in Section 2. Direct investors believe they have information about ε . After incorrect inference from y_j , each direct investor believes the randomness in firm cash flows is distributed as

$$\varepsilon \sim N\left(y_j \frac{\sigma^2}{\sigma^2 + \sigma_j^2}, \frac{\sigma^2 \sigma_j^2}{\sigma^2 + \sigma_j^2}\right).$$

Anticipating the intermediary's efforts, direct investors believe cash flows are distributed as

$$\beta a + \varepsilon | y_j \sim N\left(\beta \frac{\tau \sigma^2 \left(c\mu + \gamma A_i\right) - \beta \lambda p}{\tau \sigma^2 \left(c + \gamma\right) - \beta^2 \lambda} + y_j \frac{\sigma^2}{\sigma^2 + \sigma_j^2}, \frac{\sigma^2 \sigma_j^2}{\sigma^2 + \sigma_j^2}\right).$$

Direct investor j thus demands $\frac{E[x]-p}{Var[x]}$, which is

$$q_{j,DE}^{*}\left(y_{j},p\right) = \frac{y_{j}}{\tau\sigma_{j}^{2}} + \frac{\sigma^{2} + \sigma_{j}^{2}}{\sigma_{j}^{2}} \frac{\beta \frac{c\mu + \gamma A_{i}}{c + \gamma} - p}{\tau\sigma^{2} - \frac{\beta^{2}\lambda}{c + \gamma}}$$

The market-clearing price is given by market clearing, or $1 = \lambda q_i + \int_{j=\lambda}^1 q_j dj$. By the law of large numbers, the linear terms in y_j drop out from $\int_{j=\lambda}^1 q_j dj$, leaving price defined by

$$1 = \lambda \frac{\beta \frac{c\mu + \gamma A_i}{c + \gamma} - p}{\tau \sigma^2 - \frac{\beta^2 \lambda}{c + \gamma}} + (1 - \lambda) \frac{\sigma^2 + \sigma_j^2}{\sigma_j^2} \frac{\beta \frac{c\mu + \gamma A_i}{c + \gamma} - p}{\tau \sigma^2 - \frac{\beta^2 \lambda}{c + \gamma}}.$$

Price and demands are given in the following Proposition.

Proposition 9 When direct investors react to noisy, idiosyncratic signals, $y_j \sim N(0, \sigma_j^2)$, as if they are informative about cash flows,

1. price is given by

$$p_{DE}^{*} = \frac{\beta \left(\sigma^{2} \left(1-\lambda\right)+\sigma_{j}^{2}\right) \frac{c\mu+\gamma A_{i}}{c+\gamma}-\sigma_{j}^{2} \left(\tau \sigma^{2}-\frac{\beta^{2} \lambda}{c+\gamma}\right)}{\left(\sigma^{2} \left(1-\lambda\right)+\sigma_{j}^{2}\right)},$$

2. delegating investors hold portfolios with shares

$$q_{i,DE}^* = \frac{\sigma_j^2}{\sigma^2 \left(1 - \lambda\right) + \sigma_j^2},$$

3. and a direct investor who observes y_j holds shares

$$q_{j,DE}^{*}\left(y_{j}\right) = \frac{\sigma^{2} + \sigma_{j}^{2}}{\sigma^{2}\left(1 - \lambda\right) + \sigma_{j}^{2}} + \frac{y_{j}}{\tau\sigma_{j}^{2}}.$$

Note that $E[q_j^*] > q_i^*$, as $\sigma^2 > 0$. The direct investors demand more in expectation because they perceive lower cash flow risk. The optimal influence action and managerial

actions are:

$$m_{i,DE}^{*} = \frac{\gamma \left(A_{i}-\mu\right)}{c+\gamma} + \frac{\beta\lambda}{c+\gamma} \frac{\sigma_{j}^{2}}{\sigma^{2}\left(1-\lambda\right)+\sigma_{j}^{2}}, \text{ and}$$
$$a_{DE}^{*} = \frac{\gamma A_{i}+c\mu}{c+\gamma} + \frac{\beta\lambda}{c+\gamma} \frac{\sigma_{j}^{2}}{\sigma^{2}\left(1-\lambda\right)+\sigma_{j}^{2}}.$$

They is independent of y_j because the intermediary's demand is not affected by y_j , as the idiosyncratic noise on which direct investors make decisions does not affect price in equilibrium. Note that correlated errors, which we have not modeled here, would lead to aggregate demand shocks that would affect price and allocations in equilibrium.

Corollary 5 When direct investors react to noisy, idiosyncratic signals, $y_j \sim N(0, \sigma_j^2)$, as if they are informative about cash flows, an increase in the variance of direct investors' noise signals, σ_j^2 , leads to a decrease in the expected shares held by direct investors, $E\left[q_{j,DE}^*(y_j)\right]$, and the stock price, p_{DE}^* , and increases in the shares held by delegating investors, $q_{i,DE}^*$, the intermediary's influence effort, $m_{i,DE}^*$, managerial action, a_{DE}^* , and expected cash flows, $E\left[\beta a_{DE}^*\right]$.

Increasing σ_j^2 means that direct investors believe their signals about cash flows become worse. This effectively lowers their overconfidence, which causes them to reduce demand. Delegating investors end up holding more shares, which causes the intermediary to exert more influence efforts, increasing the manager's action and expected cash flows. However, the reduction in direct investor demand lowers stock price. Unfortunately for them, overconfidence hits direct investors twice. First, they hold too many shares because they underestimate the riskiness of cash flows. Second, their demand attenuates the incentives of the intermediary to exert effort that would increase cash flows. So, in total the direct investors hold more shares of a firm with lower cash flows.

Turning to expected utilities, direct investors anticipate investing incorrectly due to their overconfidence in the signal, y_j (think Odysseus tied to the mast). When choosing whether

to delegate or invest directly, they view y_j as a random variable to be realized in the future. Investors know that they will be susceptible to y_j in the future and can use and have correct views about the randomness in cash flows. They know that they can use delegation of investment to avoid incorrect investing due to y_j .

Proposition 10 When direct investors react to noisy, idiosyncratic signals, $y_j \sim N(0, \sigma_j^2)$, as if they are informative about cash flows, there exists an equilibrium $\lambda > 0$ such that a positive fraction of investors delegate in equilibrium.

Proposition 10 provides the result that corresponds to the intuition above. Investors can "buy" their way out of making mistakes, i.e., avoid trading decisions based on noise, by delegating their portfolios to a rational intermediary. The intermediary will in equilibrium exert costly influence efforts, and will thus require compensation that lowers delegating investors' expected utility.

Additional proofs

Proof of Proposition 10: Delegating investors have a certainty equivalent of

$$\begin{aligned} \frac{u_i^*}{\lambda} &= q_i \left(\beta \left(\frac{\gamma \left(A_i - \mu\right)}{c + \gamma} + \frac{\beta \lambda q_i}{c + \gamma} + \mu \right) - p \right) \\ &- \frac{1}{2} \tau q_i^2 \sigma^2 - \frac{\gamma}{2\lambda} \left(\frac{\gamma \left(A_i - \mu\right)}{c + \gamma} + \frac{\beta \lambda q_i}{c + \gamma} + \mu - A_i \right)^2 - \frac{c \left(\frac{\gamma \left(A_i - \mu\right)}{c + \gamma} + \frac{\beta \lambda q_i}{c + \gamma} \right)^2}{2\lambda} \\ &= -\frac{1}{2} \frac{c \gamma \left(\sigma^2 \left(1 - \lambda\right) + \sigma_j^2 \right)^2 \left(\mu - A_i \right)^2 + \lambda \sigma_j^4 \left(\beta^2 \lambda - \tau \sigma^2 \left(c + \gamma\right) \right)}{\lambda \left(\sigma^2 \left(1 - \lambda\right) + \sigma_j^2 \right)^2 \left(c + \gamma\right)}. \end{aligned}$$

At time 0 (delegation), investors view their terminal wealth as

$$W_{j} = q_{j} (x - p)$$

$$= \left(\frac{\sigma_{j}^{2} (\sigma^{2} + \tau \sigma_{j}^{2})}{\tau \sigma_{j}^{2} (\sigma^{2} (1 - \lambda) + \sigma_{j}^{2})} + (\varepsilon + \varepsilon_{j}) \frac{1}{\tau \sigma_{j}^{2}} \right) \left(\frac{\tau \sigma^{2} \sigma_{j}^{2}}{\sigma^{2} (1 - \lambda) + \sigma_{j}^{2}} + \varepsilon \right)$$

$$= \frac{\sigma^{2} \sigma_{j}^{2} (\sigma^{2} + \tau \sigma_{j}^{2})}{(\sigma^{2} (1 - \lambda) + \sigma_{j}^{2})^{2}} + \varepsilon \frac{\sigma_{j}^{2} (\sigma^{2} + \tau \sigma_{j}^{2}) + \tau \sigma^{2} \sigma_{j}^{2}}{\tau \sigma_{j}^{2} (\sigma^{2} (1 - \lambda) + \sigma_{j}^{2})}$$

$$+ \varepsilon_{j} \frac{\sigma^{2}}{\sigma^{2} (1 - \lambda) + \sigma_{j}^{2}} + (\varepsilon^{2} + \varepsilon \varepsilon_{j}) \frac{1}{\tau \sigma_{j}^{2}}.$$

Let

$$e = \begin{pmatrix} \varepsilon \\ \varepsilon_j \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \Sigma_e\right),$$

where $\Sigma_e = \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma_j^2 \end{pmatrix},$
$$w_0 = \frac{\sigma^2 \sigma_j^2 \left(\sigma^2 + \tau \sigma_j^2\right)}{\left(\sigma^2 \left(1 - \lambda\right) + \sigma_j^2\right)^2},$$

$$w_1 = \begin{pmatrix} \frac{\sigma_j^2 \left(\sigma^2 + \tau \sigma_j^2\right) + \tau \sigma^2 \sigma_j^2}{\tau \sigma_j^2 \left(\sigma^2 \left(1 - \lambda\right) + \sigma_j^2\right)} \\ \frac{\sigma^2}{\sigma^2 \left(1 - \lambda\right) + \sigma_j^2} \end{pmatrix}, \text{ and}$$

$$w_2 = \begin{pmatrix} \frac{1}{\tau \sigma_j^2} & \frac{1}{2\tau \sigma_j^2} \\ \frac{1}{2\tau \sigma_j^2} & 0 \end{pmatrix}$$

and write expected utility as $-E\left[\exp\left\{-\tau\left(w_0+w_1^Te+e^Tw_2e\right)\right\}\right]$

$$= -|I + 2\tau w_{2}\Sigma_{e}|^{-1/2} \exp\left\{-\tau w_{0} + \frac{1}{2} \left(-\tau \Sigma_{e} w_{1}\right)^{T} \left(I + 2\tau w_{2}\Sigma_{e}\right)^{-1} \Sigma_{e}^{-1} \left(-\tau \Sigma_{e} w_{1}\right)\right\}$$

$$= -\sqrt{\frac{\sigma_{j}^{2}}{\sigma^{2} + \sigma_{j}^{2}}} \exp\left\{-\tau \left(\frac{1}{2\tau} \sigma^{2} \sigma_{j}^{2} \frac{\tau^{2} \sigma_{j}^{4} - \sigma^{2} \left(\sigma^{2} \left(\tau - 1\right)^{2} - \tau^{2} \sigma_{j}^{2}\right)}{\left(\sigma^{2} + \sigma_{j}^{2}\right) \left(\sigma^{2} \left(1 - \lambda\right) + \sigma_{j}^{2}\right)^{2}}\right)\right\}$$

$$= -\exp\left\{-\tau \left(\frac{1}{2\tau} \sigma^{2} \sigma_{j}^{2} \frac{\tau^{2} \sigma_{j}^{4} - \sigma^{2} \left(\sigma^{2} \left(\tau - 1\right)^{2} - \tau^{2} \sigma_{j}^{2}\right)}{\left(\sigma^{2} + \sigma_{j}^{2}\right) \left(\sigma^{2} \left(1 - \lambda\right) + \sigma_{j}^{2}\right)^{2}} - \frac{1}{2\tau} \ln\left(\frac{\sigma_{j}^{2}}{\sigma^{2} + \sigma_{j}^{2}}\right)\right)\right\}$$

such that the certainty equivalent for the direct investors is

$$CE_{j} = \frac{1}{2\tau} \left(\sigma^{2} \sigma_{j}^{2} \frac{\tau^{2} \sigma_{j}^{4} - \sigma^{2} \left(\sigma^{2} \left(\tau - 1 \right)^{2} - \tau^{2} \sigma_{j}^{2} \right)}{\left(\sigma^{2} + \sigma_{j}^{2} \right) \left(\sigma^{2} \left(1 - \lambda \right) + \sigma_{j}^{2} \right)^{2}} - \ln \left(\frac{\sigma_{j}^{2}}{\sigma^{2} + \sigma_{j}^{2}} \right) \right)$$

Equating CE_j and CE_i/λ yields

$$\frac{1}{2\tau} \left(\sigma^2 \sigma_j^2 \frac{\tau^2 \sigma_j^4 - \sigma^2 \left(\sigma^2 \left(\tau - 1 \right)^2 - \tau^2 \sigma_j^2 \right)}{\left(\sigma^2 + \sigma_j^2 \right) \left(\sigma^2 \left(1 - \lambda \right) + \sigma_j^2 \right)^2} - \ln \left(\frac{\sigma_j^2}{\sigma^2 + \sigma_j^2} \right) \right) \\ = -\frac{1}{2} \frac{c\gamma \left(\sigma^2 \left(1 - \lambda \right) + \sigma_j^2 \right)^2 \left(\mu - A_i \right)^2 + \lambda \sigma_j^4 \left(\beta^2 \lambda - \tau \sigma^2 \left(c + \gamma \right) \right)}{\lambda \left(\sigma^2 \left(1 - \lambda \right) + \sigma_j^2 \right)^2 \left(c + \gamma \right)}.$$

This implies that the equilibrium λ is defined by

$$0 = -\sigma^{4} \ln \frac{\sigma_{j}^{2}}{\sigma^{2} + \sigma_{j}^{2}} \lambda^{3} + \left(2\sigma^{2} \left(\ln \frac{\sigma_{j}^{2}}{\sigma^{2} + \sigma_{j}^{2}} \right) \left(\sigma^{2} + \sigma_{j}^{2} \right) + \beta^{2} \tau \frac{\sigma_{j}^{4}}{c + \gamma} + c\tau \gamma \frac{\sigma^{4}}{c + \gamma} \left(\mu - A_{i} \right)^{2} \right) \lambda^{2} + \left(\frac{\sigma^{2} \sigma_{j}^{2} \frac{\sigma^{2} \left(\tau^{2} \sigma_{j}^{2} - \sigma^{2} \left(\tau - 1 \right)^{2} \right) + \tau^{2} \sigma_{j}^{4}}{\sigma^{2} + \sigma_{j}^{2}} - \tau^{2} \sigma^{2} \sigma_{j}^{4}} \right) \\ + \left(\frac{\sigma^{2} \sigma_{j}^{2} \frac{\sigma^{2} \left(\tau^{2} \sigma_{j}^{2} - \sigma^{2} \left(\tau - 1 \right)^{2} \right) + \tau^{2} \sigma_{j}^{4}}{\sigma^{2} + \sigma_{j}^{2}} - \tau^{2} \sigma^{2} \sigma_{j}^{4}} \right) \left(\sigma^{2} + \sigma_{j}^{2} \right)^{2} \right) \lambda \\ + \frac{c\tau \gamma}{c + \gamma} \left(\sigma^{2} + \sigma_{j}^{2} \right)^{2} \left(\mu - A_{i} \right)^{2}.$$

For $\lambda = 0$, the RHS is positive. For $\lambda = 1$ the RHS is

$$\frac{\sigma_j^2 \left(\begin{array}{c} \sigma_j^2 \left(\sigma^2 + \sigma_j^2\right) \left(c + \gamma\right) \left(\ln \frac{\sigma^2 + \sigma_j^2}{\sigma_j^2}\right) + \beta^2 \tau \sigma_j^2 \left(\sigma^2 + \sigma_j^2\right) \\ + c \tau \gamma \sigma_j^2 \left(\sigma^2 + \sigma_j^2\right) \left(\mu - A_i\right)^2 - \sigma^6 \left(\tau - 1\right)^2 \left(c + \gamma\right) \end{array}\right)}{\left(\sigma^2 + \sigma_j^2\right) \left(c + \gamma\right)}$$

which may or may not be negative. As $\lambda \to \infty$, the RHS of the equation goes to $-\infty$ as the coefficient on λ^3 is negative. By continuity, it must have at least one real positive root, though this root may be greater than λ . If that is the case, then the equilibrium λ will be defined by the constraint that $\lambda \in [0, 1]$ and be set to 1.

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